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THE
MOUNT VERNON
ARITHMETIC.

PART II.

VULGAR AND DECIMAL
FRACTIONS.

BY JACOB ABBOTT.

"Few rules and a great deal of practice; that is the true key to the acquisition of knowledge."

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P R E F A C E .

SUGGESTIONS TO THE TEACHER.

WHEN we assemble a class of pupils for the purpose of commencing their instruction in such a subject as fractions, it is important to remember how very abstruse and difficult it must be to minds like theirs, entirely unaccustomed as they are to generalizations and abstractions of any kind. The young have, however, an instinctive curiosity to discover what is hidden and unknown, and take an instinctive pleasure in exercising their growing powers; and they will most certainly find gratification and enjoyment in such a study as this, if the steps which we call upon them to take are short and simple enough for their embryo powers, and if we pause long enough upon each one, to make them familiar with it, before introducing another; and, above all the rest, if we are patient with them, and kind—if we speak to them in encouraging tones of voice, and if we appear satisfied with efforts, however imperfect, that are honestly and faithfully made. But if, on the other hand, the steps required of them, which our superior advancement enables us to take with ease, are a little beyond their mental powers—and if we bring in new theories and new processes to harass and perplex them, before the old are fully in possession, if we criticise harshly their unavoidable faults, if we ridicule their mistakes, and adopt the tones of stern authority or reproach in our intercourse with them—then, of course, all enjoyment on their part, in exploring the difficult ground through which we attempt to conduct them, will be out of the question. They will become disheartened and discouraged, will do nothing but what they are driven to do, and the position of the teacher, instead

of being that of a guide and leader of willing followers, will become that of an odious taskmaster, whom the bright and intelligent will dislike, and the timid and feeble will hate and fear.

The object of the Mount Vernon Arithmetic is, to present the teacher with books adapted to the former views. In the present part, the nature of fractions and fractional operations is unfolded, in a very full and thorough manner; but in so gradual an advancement, and with every new principle so illustrated and enforced with examples for practice, as will make it, the author believes, a pleasure both to teacher and class to go over the ground.

The explanations under each process, it will be observed, are not addressed to the pupil, but to the teacher;—the method of instruction being for the latter to explain fully to the class, while they are together, the principles involved in the operation which constitutes the lesson of the day. In practice, the teacher may adopt either of the two following plans, viz.: He may read over the directions himself, before coming to the class, and then, laying the book aside, explain to the pupils assembled before him the process in question, in the manner indicated:—or he may have these directions read aloud in the class by the pupils themselves—explaining each step minutely himself as the reading proceeds. The essential point is that the explanations contained in the directions should be brought fully and distinctly before the mind of every pupil, before he undertakes to perform the examples.

It is of great importance to encourage the pupils to execute the work upon the slates at all times with great neatness and care. This will not only render the teacher's labor in revising the work more easy and agreeable, but will also train the pupils to habits which will be of incalculable advantage to them, as men of business, in future life. The slates should be inspected, therefore, every day, with reference to their general appearance. Those who have made the greatest effort (not those merely who have been most successful) to arrange their work neatly and symmetrically, and to perform it in a handsome manner, should be commended. It will be an excellent plan, sometimes to place the slates together in a row, and pass them in review, so that each pupil may see the work of the others, and compare it with his own.

PREFACE.

By these and other similar modes, a high standard of accuracy, neatness and care will be formed, the attainment of which will greatly facilitate the labors of the teacher, besides being of incalculable benefit to the pupils as a branch of their education.

If this is done it will not generally be necessary for the teacher to examine personally the work of all the pupils at each successive lesson. He may call upon them to read their answers, either together or in concert, at the commencement of the exercise;—examining a few of the slates in detail each day. With a little ingenuity and tact, this will be amply sufficient to ensure very general diligence and fidelity on the part of the class, in the performance of all the work assigned.

As there will of course be a difference in the age and attainments of the different members of the class, it may happen that some will not be able to perform all the examples assigned. The teacher may, if he judges it expedient, assign to such pupils, as from age or other circumstances are not able to keep full pace with the class, a portion only of the examples in each lesson,—and accept of such portion as satisfactory, on their part, provided they have been diligent and faithful during the allotted time;—and then let all begin together with the lesson of the next day. There are usually twenty examples in each lesson; and if, in any cases, a part of these shall have been omitted, there will have been a sufficient number performed to fix in the mind with some good degree of distinctness, the nature of the process. After going through the book, those who shall thus have accomplished less than the others should review it in connexion with the next class going over the ground. In this manner pupils of somewhat different degrees of attainment can go on together; and the evil, often so distracting to the teacher, of having a great many scholars scattered along in different places in the same work, be wholly avoided.

These lessons were prepared by the author in actual practice in the instruction of classes in the institutions with which he is connected in the city of New York. He can only express his hope that other teachers may experience as much pleasure and satisfaction in guiding their pupils to a comprehension of the mysteries of fractional arithmetic, by these methods, as he has with bⁱ

The attention of the teacher is particularly invited to the exercises in the addition of columns at the commencement of the work. If the class are drilled in these columns for a short time, at every recitation, or for a longer time once a week, it will soon be found that not only their power of adding columns, but their general facility in all arithmetical operations, is increasing in a very rapid manner. Adding two or three of the sets each day will be sufficient, if a daily exercise is practised, and this will require but a few minutes at the close of each recitation.

MOUNT VERNON ARITHMETIC.

A D D I T I O N .

THE great essential, without which there can be no facility in any arithmetical operations, is the power to *add* rapidly and correctly. This, however, may be acquired very soon and very easily, by the right sort of practice; but the wrong sort of practice may be continued for a very long time, without any sensible improvement. Unfortunately, the practice which young persons are most apt to fall into, is not of the right character, and consequently very few ever attain the art of adding columns with any ease or pleasure, or with any such degree of correctness, that they can place confidence in the result.

The wrong practice above referred to, consists in this, viz., that the pupil, in going up a column, looks upon each successive number as a *collection of units*, which units he is to add, one by one, to the amount before obtained. This is not adding, it is counting. Sometimes the pupils perform this operation of counting, by drumming with the fingers on the desk—or sometimes by making marks upon the slate—and sometimes by rapidly enumerating the successive units

in the mind, dividing them into periods of two or three, to aid in keeping the reckoning. By practice, they acquire great dexterity in such processes, so that if they have, for example, 8 and 7 to add, they can run up all the steps of the ladder, 8, 9, 10, 11, 12, 13, 14, 15, very rapidly, till they get to the result 15. But it is obvious that no degree of nimbleness, in going through all these intermediate steps, can ever equal in facility the simple act of adding the 7 as a *whole*, and saying at once 8 and 7 are 15.

It is curious to observe that practice in this *counting*, however long continued, never, of itself, brings the pupil to *adding*. It carries the system of counting to as high a degree of perfection as it is capable of, and there it stops. The two methods are radically and essentially different; and to acquire the latter, it is necessary to commence right at the outset. The following tables are constructed with the design of setting the pupil upon the right road, and giving him a sufficient amount of practice to enable him to add numbers *as wholes*, in all their combinations:

The intelligent teacher will perceive that the plan consists in arranging columns, which at first contain numbers so combined that the pupil can add them, *as wholes*, with the utmost ease, so that they are under no temptation to separate them mentally, into their component units. The combinations then very gradually increase in difficulty, not so fast, however, as in any instance to endanger throwing the pupil off from the process of adding to that of counting. The whole set of tables here presented, brings, at last, all possible to view.

If the class have practiced similar tables in former parts of this work, it will be a very profitable and pleasant review for them to run over these before proceeding. If they have not practiced in this way, it is highly necessary for them to go over these tables in a thorough manner, as the power of adding numbers, at once, in all their combinations, is the essential requisite on which all arithmetical dexterity depends. Besides, in after life, in all the calculations of business, or of domestic economy, nine-tenths of the arithmetical work which is to be performed is *adding*. The power of adding, therefore, with facility and correctness, is of greater importance, in respect to its practical utility, than all the other arithmetical operations together.

The teacher is requested to pay special attention to the directions which occur occasionally, in small type. They contain suggestions which experience shows it is very important to observe, in conducting the class.

QUESTIONS TO BE ANSWERED BY THE CLASS.

How many are 5 and 5? 15 and 5? 25 and 5? 65 and 5? 75 and 5?

How many are 9 and 1? 19 and 1? 21 and 9? 31 and 9? 41 and 9? 49 and 1?

How many are 8 and 2? 28 and 2? 32 and 8? 48 and 2? 58 and 2?

How many are 7 and 3? 27 and 3? 37 and 3? 43 and 7? 23 and 7?

How many are 6 and 4? 26 and 4? 36 and 4? 84 and 6? 44 and 6?

TO THE TEACHER.

When the teacher has ascertained that the pupils are perfectly familiar with the above combinations, the latter may open their books and begin to add the unit's column of the first sum, all speaking together, thus, "*Five, ten, fifteen, twenty,*" &c., until they get to the top of the column, when they will stop with "*seventy-five; carry seven.*" They will then immediately proceed with the second column, beginning with what was to be carried, thus, "*Seven, ten, fifteen, twenty, twenty-one, thirty,*" &c., and in the same manner through all the columns. It is not necessary to write down the result, the only object being practice in going up the column.

EXAMPLES.

1.	2.	3.	4.
915,595	928,119	929,233	299,156
195,515	182,991	181,877	811,954
118,995	228,591	364,158	323,121
992,115	882,519	746,952	787,989
251,115	357,857	259,959	364,166
859,995	753,253	851,151	746,944
589,115	829,999	377,271	228,391
521,995	281,111	733,839	882,719
228,995	918,758	155,652	771,365
882,115	192,352	955,458	339,745
912,195	553,127	446,323	564,281
198,915	557,983	664,787	546,829
212,555	735,230	359,120	757,153
898,555	375,877	751,984	353,957
222,235	222,233	222,236	222,230

The teacher can easily ascertain, by observing the motions of the lips and in the class, during the exercise, whether they are the numbers in going up the column, by the in-

dependent action of their own minds. To make it certain, however, that every one actually performs the work, it is well sometimes to suspend the adding in concert, and let each pupil take a column alone, in turn. In this case, when the first pupil reaches the top of the first column, he says, "*seventy-five, carry seven.*" The second pupil immediately begins at the foot of the second column, with the "*seven,*" given him by the first, and goes on.

When a cypher occurs, no notice is to be taken of it, but the pupil passes at once to the next significant figure.

The teacher should take care that each pupil speaks in a gentle voice, and yet in a spirited and prompt manner; and everything like a sing-song tone must be avoided. No pupil must attempt to go faster than the rest, but all must keep the precise time indicated by the teacher. This time may, however, be gradually accelerated as the class acquires skill.

QUESTIONS TO BE ANSWERED BY THE CLASS.

How many are 5 and 2? 5 and 3? 13 and 5? 25 and 3? 33 and 5? 35 and 3?

5.	6.	7.	8.
525,555	552,555	525,952	125,555
225,515	252,555	222,112	113,251
315,521	322,351	333,331	232,313
115,522	532,232	525,332	232,513
222,252	122,512	532,332	233,233
222,215	232,515	223,112	223,353
221,121	233,251	232,332	122,223
225,522	112,351	432,331	325,232
112,252	323,213	222,332	324,452
212,2 5	325,323	154,112	324,222
221,121	321,521	251,332	522,222
225,522	534,511	253,332	125,223
211,222	135,115	112,112	121,213

How many are 4 and 3? 13 and 4? 23 and 4? 33 and 4? 54 and 3?
 3 and 4?

9.	10.	11.	12.
446,244	551,344	163,121	463,194
222,846	333,314	137,182	342,313
240,264	723,436	435,822	345,303
262,222	423,233	445,515	743,333
242,222	633,833	231,573	324,233
846,242	533,312	843,174	223,032
461,222	522,332	133,213	223,232
621,282	232,434	273,427	247,443
448,422	223,333	214,351	222,234
242,404	423,363	522,132	238,223
228,226	282,444	532,227	822,843
282,822	212,233	572,712	252,243
455,622	455,433	555,551	552,464

How many are 6 and 2? 6 and 3? 16 and 3? 22 and 6? 33 and 6?
 46 and 3? 53 and 6? 67 and 2?

13.	14.	15.	16.
610,535	827,791	562,127	142,507
431,412	123,511	522,133	341,311
139,296	132,518	134,758	323,421
275,212	154,121	134,532	333,371
212,212	724,442	825,273	253,318
293,182	524,532	131,315	425,736
322,353	522,135	221,632	441,132
334,353	624,372	733,434	142,932
224,332	224,348	224,434	322,262
531,571	232,321	226,312	322,822
139,547	474,423	382,351	365,526
173,205	622,623	330,143	323,534
	633,563	633,456	344,636

ADDITION.

15

How many are 7 and 2? 22 and 7? 23 and 5? 35 and 3? 24 and 4?
26 and 3? 33 and 4? 44 and 3? 53 and 6? 66 and 3?

17.	18.	19.	20.
134,315	122,116	346,321	525,511
221,221	310,115	232,301	585,540
352,553	326,365	232,218	528,132
317,223	352,721	352,271	422,123
225,823	311,212	353,117	342,314
855,112	399,815	715,172	323,544
122,252	221,162	122,221	245,233
285,532	267,522	153,216	461,222
523,212	222,465	625,123	443,411
273,312	529,223	252,231	446,212
535,711	381,812	158,243	420,183
512,334	210,343	123,323	222,153
545,435	345,415	334,343	534,422

The teacher will observe that the columns thus far have been constructed on such principles as not to present any very difficult combinations to the pupil. These examples should be practiced until all the class are perfectly familiar with all such combinations. It is very important, also, to make sure that each pupil actually adds for himself, instead of catching the sound from the voices of the others. For this purpose, it will be best to review these examples, in the following manner. Let each pupil add one column alone, giving the amount which is to be carried, to the next, who will proceed with the next column. When the whole sum is added in this manner, which will be done by six pupils, there being six columns in each sum, the whole class may repeat the process in concert; and then the next example may be taken in the same manner. All the class should attend carefully while each individual pupil is adding, and say "*wrong*," in a gentle voice, when he makes a mistake.

In the following examples, more difficult combinations are introduced, though only one new combination is brought into each two sums; and these combinations are illustrated by the questions which precede them. A key to the new combination of each sum will also be found in the two lowest figures of the units column.

How many are 9 and 2? 29 and 2? 8 and 3? 28 and 3? 39 and 2?
49 and 2? 48 and 3? 58 and 3? 19 and 2? 38 and 3? 59 and 2?

21.	22.	23.	24.
221,325	931,921	922,393	491,591
888,374	261,224	221,133	533,523
212,222	426,355	822,113	215,214
312,445	412,329	325,563	222,373
342,523	239,312	133,332	452,533
355,222	352,971	337,275	396,177
929,321	423,226	332,523	923,323
252,542	223,312	288,312	337,391
424,232	443,221	313,222	333,312
424,812	524,244	327,322	336,283
212,221	256,224	322,221	292,231
411,122	252,222	353,333	233,623
134,439	253,439	352,348	232,428

How many are 9 and 3? 29 and 3? 39 and 3? 49 and 3? 7 and 4?
27 and 4? 37 and 4? 47 and 4? 39 and 2? 37 and 4? 47 and 4?

25.	26.	27.	28.
339,215	283,358	444,031	832,958
470,633	732,230	554,928	382,453
352,389	242,233	443,354	832,736
832,228	255,336	763,897	322,873
337,223	418,933	346,239	753,384
235,520	373,331	579,714	323,435
233,827	833,792	393,367	893,544
533,333	376,332	842,749	243,857
988,813	229,825	412,473	714,398
333,223	553,842	542,224	354,643
434,941	234,326	222,363	742,318
464,333	234,433	221,324	310,954
	233,529	233,337	422,447

ADDITION.

17

How many are 8 and 4? 18 and 4? 28 and 4? 38 and 4? 48 and 4?
9 and 4? 29 and 4? 39 and 4? 49 and 4? 98 and 4? 99 and 4?

29.	30.	31.	32.
154,253	917,193	772,145	352,828
192,453	483,423	432,255	254,344
624,233	742,689	354,437	454,785
474,452	276,328	337,484	781,444
338,135	633,294	434,449	346,756
598,574	457,592	785,540	854,324
693,442	244,326	443,748	232,673
433,622	236,372	654,493	685,443
423,432	254,332	345,447	443,654
423,334	388,372	861,553	914,233
228,586	948,242	423,346	802,823
243,144	283,884	726,774	203,224
224,348	134,258	434,339	532,519

How many are 6 and 5? 16 and 5? 36 and 5? 46 and 5? 7 and 5?
27 and 5? 37 and 5? 47 and 5? 26 and 5? 36 and 5? 87 and 5?

33.	34.	35.	36.
844,157	744,185	335,485	544,252
332,264	485,242	335,253	447,453
745,046	625,644	474,267	554,883
526,153	053,442	754,238	468,222
444,558	444,587	425,455	449,867
255,563	886,352	755,552	245,543
734,446	544,247	335,473	243,476
433,454	647,723	356,535	524,434
793,658	234,496	354,551	825,468
443,344	698,334	247,752	254,432
445,636	448,467	843,972	929,826
436,495	874,235	497,335	363,395
345,236	333,236	244,227	252,257

How many are 8 and 5? 18 and 5? 28 and 5? 38 and 5? 9 and 5?
29 and 5? 39 and 5? 48 and 5? 59 and 5? 98 and 5? 99 and 5?

37.	38.	39.	40.
512,555	717,500	224,850	502,351
625,572	410,201	723,370	502,528
428,245	451,444	352,350	722,655
352,431	454,631	772,541	543,562
578,432	465,455	532,648	852,342
151,552	445,625	655,572	162,543
328,515	436,525	251,447	354,853
322,755	555,725	223,523	365,423
427,755	585,222	272,926	556,533
323,542	645,322	224,754	465,352
356,853	827,351	263,535	556,664
544,285	162,455	229,755	862,355
<u>353,258</u>	<u>542,568</u>	<u>353,549</u>	<u>562,339</u>

How many are 5 and 6? 15 and 6? 25 and 6? 35 and 6? 45 and 6?
6 and 6? 26 and 6? 56 and 6? 76 and 6? 96 and 6? 96 and 6?

41.	42.	43.	44.
545,460	410,251	663,780	632,222
446,451	610,202	244,312	232,827
543,558	356,652	562,762	234,465
643,554	745,267	262,436	244,336
542,293	127,752	332,368	444,354
236,573	155,377	552,356	544,365
336,456	246,656	942,566	433,365
548,274	224,472	781,438	532,364
244,266	283,275	500,566	434,384
247,613	255,353	600,756	534,368
244,321	589,544	410,568	424,323
266,526	529,856	830,756	524,326
	<u>515,345</u>	<u>532,336</u>	<u>424,356</u>

ADDITION.

19

How many are 7 and 6 ? 17 and 6 ? 27 and 6 ? 37 and 6 ? 107 and 6 ?
8 and 6 ? 18 and 6 ? 28 and 6 ? 38 and 6 ? 48 and 6 ? 58 and 6 ?

45.	46.	47.	48.
230,953	481,623	663,823	625,551
230,013	452,402	616,501	662,610
767,436	632,472	246,242	665,535
554,264	265,656	543,666	656,755
763,335	751,557	482,654	655,444
553,254	653,537	665,456	743,826
454,555	476,326	634,553	545,357
453,363	232,521	704,432	828,723
443,633	265,323	366,323	324,254
475,366	743,256	777,352	378,455
288,674	267,454	388,634	323,446
222,516	141,216	743,736	327,266
253,647	134,357	524,528	324,328

How many are 9 and 6 ? 19 and 6 ? 39 and 6 ? 89 and 6 ? 99 and 6 ?
4 and 7 ? 14 and 7 ? 34 and 7 ? 54 and 7 ? 69 and 6 ? 64 and 7 ?

49.	50.	51.	52.
530,245	251,491	514,953	223,524
236,635	666,567	457,337	334,543
565,732	484,233	724,763	333,366
674,553	735,562	312,654	326,249
376,666	665,435	466,575	323,627
644,564	346,164	711,433	662,893
535,665	332,337	334,373	663,777
633,625	363,364	376,212	637,436
653,356	352,634	637,733	454,642
536,629	362,765	537,667	766,322
346,645	888,673	233,823	826,483
232,646	333,636	332,237	128,737
523,639	223,139	362,254	624,534

How many are 5 and 7? 15 and 7? 25 and 7? 35 and 7? 6 and 7?
16 and 7? 36 and 7? 46 and 7? 45 and 7? 36 and 7? 36 and 7?

53.	54.	55.	56.
312,723	166,712	134,754	227,272
222,123	153,307	732,431	263,755
222,452	486,565	573,622	445,251
452,637	638,453	543,657	456,577
427,723	131,346	777,426	423,575
662,577	737,666	534,761	446,537
254,833	247,637	876,376	165,768
336,555	661,753	553,777	346,347
555,251	726,215	727,374	333,736
334,572	224,727	644,726	346,441
335,812	536,338	466,623	855,346
538,567	636,327	266,127	126,637
344,235	666,245	263,346	423,736

How many are 7 and 7? 17 and 7? 27 and 7? 37 and 7? 8 and 7?
18 and 7? 28 and 7? 38 and 7? 37 and 7? 98 and 7? 97 and 7?

57.	58.	59.	60.
221,244	135,275	152,217	422,435
364,472	416,232	153,716	162,365
675,677	518,454	377,322	226,972
762,613	553,757	773,615	254,243
627,657	574,261	413,437	674,273
683,273	647,543	772,533	543,272
614,757	567,373	445,527	477,715
754,442	632,333	447,433	573,336
773,743	560,273	241,677	225,736
133,874	634,232	227,338	532,873
972,874	432,574	761,775	844,535
172,217	232,577	829,887	232,637
252,437	525,357	443,538	236,348

ADDITION.

31

How many are 9 and 7? 19 and 7? 29 and 7? 3 and 8? 43 and 8?
53 and 8? 9 and 7? 59 and 7? 53 and 8? 63 and 8? 69 and 7?

61.	62.	63.	64.
445,472	216,572	232,475	474,782
562,532	606,662	640,338	533,848
625,717	202,332	752,536	523,453
326,705	513,626	637,432	748,384
468,612	734,364	368,225	513,223
667,662	627,471	375,773	372,332
466,335	624,453	294,531	332,448
336,246	663,736	163,262	445,763
753,266	667,453	333,747	722,623
425,223	188,337	727,543	324,832
425,374	138,673	538,635	835,124
856,577	683,937	471,128	128,438
737,679	732,449	334,753	747,343

How many are 4 and 6? 34 and 6? 5 and 8? 15 and 9? 34 and 8?
24 and 8? 74 and 8? 75 and 8? 65 and 8? 64 and 8? 75 and 8?

65.	66.	67.	68.
156,724	158,487	184,781	267,763
613,267	382,888	338,264	422,147
524,145	884,562	765,762	667,737
847,537	746,726	732,466	641,633
413,826	728,267	447,678	647,624
325,368	581,482	455,778	774,654
255,284	264,682	457,624	744,886
434,563	672,315	273,738	444,435
537,417	872,751	373,353	343,775
324,354	635,731	575,282	585,374
283,783	848,886	237,748	762,736
734,748	943,458	617,568	765,808
472,354	432,444	447,765	757,505

How many are 6 and 8? 16 and 8? 26 and 8? 36 and 8? 7 and 8?
17 and 8? 36 and 8? 47 and 8? 56 and 8? 77 and 8? 97 and 8?

69.	70.	71.	72.
776,445	513,278	157,423	237,335
218,656	402,558	373,828	753,366
242,386	875,535	745,177	776,423
748,324	887,247	258,563	744,837
982,568	878,778	527,575	783,283
747,745	447,757	442,383	425,774
482,277	447,777	677,348	579,872
513,378	494,777	332,454	378,325
753,876	557,268	958,627	883,788
253,782	735,664	217,327	742,734
665,634	728,834	776,375	664,244
733,338	783,328	356,238	956,848
572,326	742,476	276,877	321,457

How many are 8 and 8? 18 and 8? 28 and 8? 38 and 8? 58 and 8?
9 and 8? 19 and 8? 39 and 8? 49 and 8? 59 and 8? 89 and 8?

73.	74.	75.	76.
284,332	148,124	128,128	813,161
180,458	172,184	181,216	212,321
811,442	289,831	734,342	485,246
421,229	811,271	261,222	131,142
419,343	514,942	834,281	326,331
282,214	123,148	121,851	253,635
114,311	481,812	341,238	124,414
113,183	142,233	248,522	742,132
882,128	268,963	121,318	314,218
229,931	154,122	383,211	531,711
863,121	614,192	116,881	221,281
216,898	141,718	231,238	328,128
114,328	234,328	214,319	316,239

ADDITION.

33

How many are 2 and 9? 12 and 9? 22 and 9? 32 and 9? 42 and 9?
3 and 9? 13 and 9? 23 and 9? 34 and 9? 53 and 9? 62 and 9?

77.	78.	79.	80.
838,484	834,391	791,955	186,582
631,492	138,297	891,981	266,596
669,428	797,619	838,977	368,894
371,773	799,432	729,489	447,996
692,977	737,771	829,664	553,399
817,143	286,997	859,665	642,944
794,942	697,613	889,523	777,742
644,164	839,527	283,979	877,737
667,222	824,318	888,473	943,732
787,921	877,772	883,275	145,838
875,977	723,991	885,772	260,838
873,999	766,139	382,389	360,899
434,472	745,442	883,863	765,873

How many are 4 and 9? 14 and 9? 24 and 9? 34 and 9? 5 and 9?
15 and 9? 25 and 9? 35 and 9? 55 and 9? 64 and 9? 76 and 9?

81.	82.	83.	84.
751,154	359,776	995,296	857,115
247,138	169,255	998,315	854,018
777,742	449,485	929,698	744,218
735,796	759,162	694,717	777,889
835,819	622,977	337,787	264,989
897,847	559,158	656,537	656,389
338,867	533,944	666,194	884,899
999,795	458,124	477,244	554,399
377,416	723,975	498,233	757,796
766,446	932,737	415,532	397,997
644,563	972,489	793,893	642,891
388,899	365,779	973,819	732,679
388,574	383,874	587,775	878,785

How many are 6 and 9? 16 and 9? 26 and 9? 36 and 9? 7 and 9?
17 and 9? 27 and 9? 37 and 9? 46 and 9? 66 and 9? 88 and 9?

85.	86.	87.	88.
728,463	684,595	937,239	199,139
775,837	287,497	691,451	171,936
733,855	987,777	213,319	913,722
622,377	948,886	299,927	329,293
555,766	744,984	127,191	172,219
778,588	777,875	932,912	345,993
798,877	582,787	132,291	292,174
577,477	388,887	419,359	272,922
329,995	784,979	122,431	929,222
778,796	999,926	293,229	223,499
227,324	377,945	213,911	594,271
843,499	775,939	192,199	211,229
361,836	884,436	122,117	214,127

How many are 8 and 9? 18 and 9? 28 and 9? 38 and 9? 9 and 9?
19 and 9? 29 and 9? 38 and 9? 49 and 9? 57 and 9? 69 and 9?

89.	90.	91.	92.
969,249	431,745	141,913	611,164
362,937	499,214	911,162	499,312
729,291	322,521	218,943	221,491
312,923	281,311	284,311	221,229
322,699	923,189	616,399	631,435
195,131	234,322	111,431	137,151
324,141	754,899	923,119	311,914
426,232	626,213	164,922	493,192
463,994	434,984	942,246	212,211
153,142	211,121	212,612	194,317
224,741	645,462	858,291	914,481
291,119	131,419	322,129	426,239
	<u>412,348</u>	<u>232,329</u>	<u>125,219</u>

In the following examples the pupil is to add each set of numbers inclosed in brackets as one number, they being such that he can easily perceive the amount thus included, by a glance of the eye. Thus, in the first column, he will say, "*three, thirteen, nineteen, twenty-nine,*" &c.

93.

1	2	1	1	5	5
3	4	8	1	5	5
7	4	2	8	5	2
4	1	2	1	2	5
6	2	6	9	8	4
3	7	7	3	4	6
7	5	3	7	1	3
4	5	9	1	9	3
2	3	3	3	2	7
2	2	8	3	5	6
7	5	3	3	7	5
6	4	7	7	3	5
4	7	5	7	4	3

94.

1	7	5	7	1	2
2	3	4	2	3	1
4	8	2	9	4	9
6	9	1	4	2	8
5	2	3	3	1	3
3	1	4	2	5	4
2	4	3	1	9	2
1	3	6	3	4	1
9	2	4	6	3	8
3	5	7	4	2	0
3	7	2	5	1	
3	1	2	9	6	2
1	6	1	1	4	3

95.

3	2	2	3	3	2
2	3	4	2	0	1
7	0	8	1	0	7
8	0	3	4	0	9
1	1	7	6	4	4
9	9	8	3	2	1
3	2	9	2	4	5
2	1	4	5	5	3
4	6	5	8	7	2
1	4	2	2	8	4
5	8	2	7	2	3
6	3	8	2	1	2
4	7	9	1	3	1

96.

3	4	2	1	3	5
2	2	4	2	3	6
7	1	0	0	0	1
2	3	2	0	1	7
5	7	8	1	9	1
1	3	2	1	9	1
2	4	6	4	2	1
3	3	2	1	4	2
4	6	4	3	2	1
2	4	2	1	8	2
1	4	2	1	4	3
2	2	2	8	1	3
3	4	8	1	2	1

97.

$$\begin{array}{r}
 1 \} 3 \ 5 \ 9 \} 7 \ 6 \\
 3 \} 7 \ 2 \} 1 \ 3 \ 1 \\
 2 \ 3 \} 2 \} 2 \ 4 \ 1 \} \\
 2 \ 8 \ 1 \ 8 \} 2 \ 9 \} \\
 3 \} 7 \ 2 \ 3 \ 7 \ 2 \\
 3 \} 2 \ 2 \} 1 \ 5 \ 7 \\
 1 \ 3 \} 5 \ 3 \ 2 \ 5 \} \\
 3 \ 5 \} 6 \ 1 \ 4 \ 5 \} \\
 6 \ 5 \} 1 \} 2 \ 7 \ 8 \\
 1 \} 6 \ 1 \} 2 \ 1 \ 4 \\
 1 \ 2 \} 1 \} 1 \ 1 \ 8 \\
 1 \ 2 \} 1 \} 6 \ 1 \ 3 \} \\
 1 \} 7 \ 5 \ 9 \ 4 \ 7 \}
 \end{array}$$

98.

$$\begin{array}{r}
 2 \} 1 \} 4 \ 6 \ 1 \} 9 \} \\
 2 \} 1 \} 9 \ 8 \ 7 \} 1 \} \\
 3 \} 2 \ 1 \} 6 \ 4 \ 1 \} \\
 1 \} 8 \ 1 \} 4 \ 0 \ 1 \} \\
 6 \} 4 \ 8 \} 5 \ 1 \} 5 \} \\
 4 \} 1 \} 7 \ 1 \} 1 \} 5 \} \\
 3 \} 7 \ 4 \ 1 \} 1 \} 3 \} \\
 7 \} 5 \ 3 \} 0 \ 0 \} 7 \} \\
 1 \} 4 \ 3 \} 6 \ 2 \} 8 \\
 1 \} 1 \} 3 \} 2 \ 1 \} 4 \\
 1 \} 2 \ 1 \} 1 \} 4 \ 3 \\
 7 \ 6 \ 1 \} 1 \} 7 \ 3 \} \\
 8 \ 7 \ 4 \ 7 \ 4 \ 2 \}
 \end{array}$$

99.

$$\begin{array}{r}
 2 \ 3 \ 1 \ 2 \ 1 \} 1 \\
 1 \ 4 \} 0 \ 1 \ 0 \} 7 \} \\
 9 \} 1 \} 3 \ 3 \ 1 \} 3 \} \\
 1 \} 1 \} 3 \ 1 \} 7 \ 1 \\
 2 \ 8 \ 1 \} 1 \} 4 \ 0 \\
 8 \} 4 \ 1 \} 0 \} 3 \ 7 \\
 2 \} 0 \ 1 \} 1 \} 3 \ 7 \\
 2 \} 1 \} 2 \ 1 \} 3 \ 1 \} \\
 1 \} 7 \ 1 \} 5 \ 7 \ 1 \} \\
 7 \} 8 \ 7 \} 3 \ 1 \} 3 \} \\
 5 \ 4 \ 1 \} 1 \} 1 \ 2 \\
 7 \} 8 \ 1 \} 5 \ 8 \} 2 \} \\
 3 \} 1 \} 8 \ 4 \ 1 \} 2 \} \\
 8 \ 1 \} 3 \} 4 \ 1 \} 9 \\
 7 \ 4 \ 3 \} 4 \ 3 \ 5
 \end{array}$$

100.

$$\begin{array}{r}
 2 \ 5 \ 3 \} 8 \} 1 \} 1 \} \\
 1 \ 6 \ 7 \} 2 \} 9 \} 0 \} \\
 6 \} 3 \} 4 \ 1 \ 2 \ 1 \} \\
 1 \} 6 \} 3 \} 5 \ 5 \ 3 \\
 5 \} 5 \ 3 \} 1 \ 1 \} 4 \\
 5 \} 2 \} 9 \ 3 \} 2 \} 1 \} \\
 4 \ 3 \} 1 \} 3 \} 0 \} 1 \} \\
 3 \} 9 \ 9 \} 1 \} 1 \} 1 \} \\
 3 \} 2 \ 4 \ 3 \} 3 \ 3 \} \\
 1 \ 3 \} 3 \ 9 \ 1 \} 7 \} \\
 3 \} 3 \} 2 \ 8 \ 0 \} 5 \} \\
 3 \} 3 \} 2 \ 2 \} 2 \} 5 \} \\
 3 \} 1 \} 4 \ 1 \} 3 \ 1 \} \\
 6 \ 9 \} 3 \ 3 \ 8 \ 1 \} \\
 7 \ 3 \ 8 \ 3 \ 5 \ 8 \}
 \end{array}$$

I. GENERAL NATURE OF FRACTIONS.

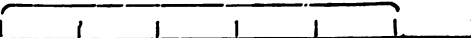
LESSON I.

Let the teacher draw a line upon the black-board, requesting the class to do the same upon their slates, and then divide it into four parts—first halving the whole line, and then again halving each part, the class imitating the example with their lines. Then let him draw a *vinculum*, that is, a line with the ends curved downwards, over a portion of the line, so as to cover three of the parts, thus—

No. 1. 

Then let him explain to the class that the whole number of parts into which the line is divided being *four*, and the number cut off by the vinculum being *three*, the fraction *three-fourths* is represented by the illustration.

Then let the teacher draw other examples, asking the class what fraction is represented, as follows :—

No. 2. 

No. 3. 

No. 4. 

No. 5. $\frac{\quad}{\quad}$

And so on until the class are perfectly familiar with the nature of a fraction.

Let the teacher next explain that such fractions are expressed by two figures with a line between them, and that the number below the line denotes the number of parts into which the line is divided, and is called the *denominator*, and that the one above the line denotes the number of parts included under the vinculum, and is called the *numerator*. Then let him ask them the following questions, to be answered by the class individually or in concert, until all are perfectly familiar with the subject.

QUESTIONS.

Into how many parts is line No. 1 divided? How many parts are cut off by the vinculum? What then is the fraction represented? In No. 2? In No. 3? In No. 4? In No. 5?

What is the fraction represented in the first case?
Ans. $\frac{1}{2}$. In writing that fraction which figure do you place above the line? What is it called? What is the figure to be placed below the line? What is it called?

What fraction is represented in No. 2? What is the numerator? What the denominator? In No. 3? In No. 4? In No. 5?

The teacher is to repeat these questions in respect to other lines drawn by himself upon the black-board, directing the attention of the class to them with the pointer, until they can answer with facility and correctness. Then he may assign the following exercises to be performed by the class at their seats: the work upon the specimens arranged in accordance with the specimen given.

EXAMPLES.

1. Represent upon the slate the fraction *three-fourths* in words, in figures, and in a divided line.

Specimen of the Work upon the Slate.

(1.) Three-fourths. | $\frac{3}{4}$ | 

The same with the following examples, which are to be performed by the class at their seats, and the work to be arranged neatly, as above, in three columns, extending down the slate as far as may be necessary. The number of the example should always accompany it upon the slate, and it should be enclosed in a parenthesis, as in the specimen, to distinguish it from the figures pertaining to the operation.

- | | |
|--------------------|-----------------------|
| 2. Four-fifths. | 11. One-third. |
| 3. Three-sevenths. | 12. Five-twentieths. |
| 4. One-half. | 13. Seven-twentieths. |
| 5. Two-fifths. | 14. Four-eighths. |
| 6. One-eighth. | 15. Three-sevenths. |
| 7. Two-eighths. | 16. Five-sixths. |
| 8. Three-sevenths. | 17. Eleven-twelfths. |
| 9. One-ninth. | 18. Six-twelfths. |
| 10. Five-ninths. | 19. Two-quarters. |

When the work, as above directed, is presented for examination by the teacher at the next meeting of the class, particular attention should be paid to the neatness and order in which it is arranged, and in the case of the older and more advanced scholars, to the accuracy of the divisions of the lines, though these divisions should be made by the eye alone. If any of the pupils are less advanced than the rest, so that they work more slowly, they may perform

only such a portion of the examples assigned, as the time allotted them allows, provided that the work which they do perform is neatly and carefully executed; while those who can perform more than those above assigned, are to add others of their own invention, so as to occupy all the time allotted for the work.

LESSON II.

Explain to the class that to *denominate* means to *name*, and that the reason why the number below the line is called the *denominator*, is that it expresses the *name* of the *kind of parts* into which the line is divided.

QUESTIONS.

If a line is divided into five parts, what is the name of the parts? *Ans.* Fifths.

If a number is divided into six parts, what is the denomination of the parts? *Ans.* Sixths.

And what figure would be a proper *denominator*, to express this denomination? *Ans.* 6.

If a sum of money is divided into four parts what is the denomination? Fourths. And what would be the *denominator* of the fraction expressing fourths? *Ans.* 4.

What is the denomination of parts in this fraction $\frac{2}{3}$? The denominator? $\frac{3}{4}$? $\frac{5}{8}$? $\frac{7}{16}$? $\frac{1}{2}$?

Explain to the class that to *numerate* means to number—and that the reason why the figure above the line is called the numerator, is because it denotes the number of the parts which the fraction is intended to express; and that thus the denominator expresses the *name of the parts*, and the numerator the number of them.

How many parts are denoted by this fraction, seven-tenths? *Ans.* Seven. What is the name of the parts? *Ans.* Tenths.

What then is the numerator? What is the denominator?

What is the name of the parts in this fraction, $\frac{3}{4}$? The number? In $\frac{1}{2}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{6}{7}$? $\frac{7}{8}$? $\frac{8}{9}$? $\frac{9}{10}$? $\frac{10}{11}$? $\frac{11}{12}$? $\frac{12}{13}$? $\frac{13}{14}$? $\frac{14}{15}$?

What is the numerator in this fraction, $\frac{3}{4}$? The denominator? In $\frac{1}{2}$? In $\frac{2}{3}$? $\frac{3}{4}$? $\frac{4}{5}$?

Let the pupil divide the surface of the slate, upon one side, into five columns, by perpendicular lines, and write heads to the columns as below; and then fill them in the manner here exemplified,—the teacher dictating the fractions to be entered in the first column, and the pupils entering the numerator, denominator, &c. in the proper columns.

Work upon the Slate.

	Fraction.	Numerator.	Denominator.	Name of parts.	Number of parts.
(20)	$\frac{1}{4}$	1	4	Fourths	One
(21)	$\frac{3}{5}$	3	5	Fifths	Three
(22)	$\frac{7}{10}$	7	10	Tenths	Seven

When this is well understood the class may go to their seats and continue the table above commenced by recording, in a very neat and careful manner, an analysis of the following fractions.

EXAMPLES.

23. $\frac{3}{2}$	29. $\frac{1}{2}$
24. $\frac{7}{4}$	30. $\frac{1}{4}$
25. $\frac{9}{16}$	31. $\frac{2}{3}$
26. $\frac{1}{16}$	32. $\frac{1}{3}$
27. $\frac{4}{5}$	33. $\frac{1}{5}$
28. $\frac{1}{5}$	34. $\frac{2}{5}$

LESSON III.

Explain to the class that a fraction may be doubled by doubling the *number of the parts* expressed by it, that is, by doubling the numerator. Thus $\frac{2}{3}$ doubled becomes $\frac{4}{3}$. This may be expressed on the blackboard thus :



In the same manner let the pupils double the following fractions, at their seats, representing both values neatly upon the slate as above.

EXAMPLES.

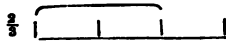
35. $\frac{2}{3}$	41. $\frac{1}{15}$
36. $\frac{1}{16}$	42. $\frac{3}{16}$
37. $\frac{2}{12}$	43. $\frac{3}{4}$
38. $\frac{3}{5}$	44. $\frac{1}{9}$
39. $\frac{6}{8}$	45. $\frac{2}{8}$
40. $\frac{7}{8}$	46. $\frac{1}{8}$

As the class advances, the work upon the slate must be inspected with reference to the neatness and beauty of the work, and the ac-

curacy of the linear divisions. The pupils must be taught how to divide the lines in the easiest manner. Thus, to make six divisions, they divide the line first into halves, and then each half into thirds. To make seven divisions, they may first make the line a little shorter than it is ultimately designed to be, and divide it into six parts as above, and then lengthen the line one division, which will make seven. To represent nineteen divisions, make the line at first a little shorter than it is intended to be, and divide it into sixteenths, by repeated subdivisions, and then lengthen the line sufficiently for three additional parts. A great deal of practice of this kind will be found extremely useful, in making the pupil perfectly familiar with the real import of fractional expressions, as well as giving him manual skill in executing such work in a neat and systematic manner, which will be of great importance in future life. If those who are more advanced and skillful than the rest, have time to perform more examples than those given in the book, they are not to be idle, but to add others of their own invention, so as to present a slate as well and as carefully filled as they are able to do, in the allotted time.

LESSON IV.

Explain to the pupil that a fraction may be doubled by doubling the *magnitude* of the parts, as well as by doubling the number. Thus, $\frac{2}{3}$ is double of $\frac{1}{3}$ because a third is twice as much as a sixth, thus—



QUESTIONS.

What part of a line is twice as great as a sixth?
Ans. A third. What part is twice as great as a tenth?
 As a twelfth? A fourth? An eighth? A hundredth?
 A fiftieth?

Double *three-tenths* by increasing the magnitude of the parts. *Ans.* $\frac{3}{5}$. Double $\frac{1}{4}$ by doubling the magnitude of the parts, leaving the number the same. Double $\frac{5}{10}$. $\frac{7}{10}$. $\frac{3}{5}$. $\frac{1}{2}$.

Let the pupils observe that in these cases the operation of doubling the fraction is performed by *dividing the denominator*, as, in the former case, it was done by *multiplying the numerator*.

47. Double the fraction $\frac{1}{4}$ by *dividing the denominator*, and represent both values upon the slate, as in the former lessons, thus—

Work upon the Slate.

(47.) $\frac{1}{4}$  doubled is $\frac{1}{2}$ 

EXAMPLES.

48.	$\frac{1}{8}$	54.	$\frac{2}{10}$
49.	$\frac{1}{6}$	55.	$\frac{3}{12}$
50.	$\frac{3}{10}$	56.	$\frac{4}{10}$
51.	$\frac{1}{8}$	57.	$\frac{5}{10}$
52.	$\frac{1}{8}$	58.	$\frac{1}{10}$
53.	$\frac{1}{10}$	59.	$\frac{1}{4}$

LESSON V.

QUESTIONS.

In the fraction $\frac{1}{4}$, what is the *denomination* of the
 parts. What is the *number* of the

parts? *Ans.* Two. Double the fraction by doubling the *magnitude* of the parts, while you leave the number the same. *Ans.* $\frac{2}{3}$, two halves. Double the fraction by doubling the *number* of the parts, the magnitude remaining the same. *Ans.* $\frac{4}{3}$, four quarters.

In doubling the number of the parts, do you operate upon the numerator or denominator? Do you multiply or divide it? In doubling the magnitude of the parts, do you operate on the numerator or denominator? Do you multiply or divide it?

Continue such questions and illustrations in the class until all perfectly understand that a fraction may be doubled either by *doubling the numerator*, while the denominator remains unchanged, or *halving the denominator*, while the numerator remains unchanged.

Then assign the following examples, to be performed by both methods after the class have retired to their seats. The signs used in the specimen of the manner in which the work should be performed on the slate, as given below, should first be fully explained.

60. Multiply $\frac{3}{4}$ by 2, in both the modes explained above.

Work upon the Slate.

$$(60.) \quad \frac{3}{4} \times 2 = \frac{6}{4} = \frac{3}{2}$$

EXAMPLES.

- | | |
|-----------------------------|-----------------------------|
| 61. $\frac{7}{8} \times 2$ | 66. $\frac{3}{16} \times 2$ |
| 62. $\frac{2}{3} \times 2$ | 67. $\frac{7}{16} \times 2$ |
| 63. $\frac{1}{16} \times 2$ | 68. $\frac{1}{16} \times 2$ |
| 64. $\frac{3}{8} \times 2$ | 69. $\frac{5}{16} \times 2$ |
| 65. $\frac{5}{16} \times 2$ | 70. $\frac{1}{16} \times 2$ |

LESSON VI.

Explain to the pupil that fractions may be multiplied by 3 or any other number on the same principles as above, either by multiplying the numerator or dividing the denominator, in case the denominator is divisible by the number in question. This may be made clear to their minds by a conversation like the following :

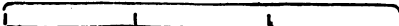
Teacher. Write the fraction $\frac{3}{9}$ upon the slate. Make the sign for multiplication on the right hand and beyond it the figure 3. Now read what you have written. *Class.* Three-ninths multiplied by three. *Teacher.* Now make the equation sign beyond, and write the answer beyond it; and each of you may rise as soon as you have finished the work.

Then as the pupils rise, the teacher inspects their slates, to see not only if the correct answer is obtained, but if the figures and characters are made well, and the work neatly arranged. Those who hesitate, or are at a loss precisely how to perform the operation, are to be encouraged and assisted until all are up, and then the whole class are to read what they have written, together, thus :

Three-ninths multiplied by three, equals nine-ninths. *Teacher.* Write another equation sign beyond the last fraction, $\frac{3}{9}$. Consider now whether you can multiply the original fraction by dividing the denominator. What is the original fraction? *Ans.* $\frac{3}{9}$. What is the denominator? *Ans.* Nine. What is the denomination of the parts? *Ans.* Ninths. Can you divide 9 by 3? What will be the result? What then will the original fraction become? *Ans.* $\frac{3}{3}$. Is three-thirds three times as much as three-ninths? Then write three-thirds beyond the equation mark and read the whole line. *Class.* Three-ninths multiplied by three, equals, &c.

The teacher may illustrate this upon the blackboard thus :

3 times $\frac{3}{9}$ is $\frac{3}{3}$ thus, 

Or it is $\frac{3}{3}$ thus, 

By such a diagram it will be clear to the class that three times three-ninths may be considered as nine-ninths, or as three-thirds. These illustrations, varied according to the ingenuity of the teacher and the capacity of the class, should be continued, not merely until all the class understand the subject, but until their minds have become familiarized to the consideration of it in all its lights. This kind of familiarity with the essential nature of fractions, obtained at the outset, will greatly facilitate all subsequent progress.

After this proceed to assign the following examples to be performed at the seats,—the work to be arranged thus :

Work upon the Slate.

$$(71.) \quad \frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$$

$$(72.) \quad \frac{1}{8} \times 3 = \frac{3}{8} = \frac{3}{8}$$

EXAMPLES.

$$71. \quad \frac{3}{8} \times 2$$

$$78. \quad \frac{3}{18} \times 5$$

$$72. \quad \frac{1}{8} \times 3$$

$$79. \quad \frac{5}{18} \times 2$$

$$73. \quad \frac{6}{18} \times 2$$

$$80. \quad \frac{6}{18} \times 5$$

$$74. \quad \frac{6}{18} \times 4$$

$$81. \quad \frac{3}{18} \times 4$$

$$75. \quad \frac{3}{8} \times 3$$

$$82. \quad \frac{3}{18} \times 3$$

$$76. \quad \frac{1}{18} \times 5$$

$$83. \quad \frac{3}{18} \times 6$$

$$77. \quad \frac{2}{18} \times 5$$

$$84. \quad \frac{3}{18} \times 2$$

When the above examples are brought to the class, after being performed by the pupils at their seats, it is necessary to make each individual thoroughly familiar with the nature and import of the operations they have performed upon each fraction, which may best be done by having them recite their work in answer to such questions as the following, the pupils sometimes being called upon to answer individually and sometimes in concert.

What is the first fraction given? *Ans.* Three-eighths. What kind of parts are denoted by the fraction $\frac{3}{8}$? *Ans.* Eighths. How many are denoted? Three. What were you to multiply the three-eighths by? Two. What is the first form of the answer? Six-eighths. In this case is the *number* or the *magnitude* of the parts altered? The number. What is the second form of the answer? Three-fourths. In this case is the number or the magnitude of the parts multiplied? The magnitude. What were the parts before? Eighths. What are they now? Fourths. Which is greatest, a fourth or an eighth? A fourth. How much greater? Twice as great, &c.

This kind of exercise should be continued through all the examples. It will be very effectual in opening the minds of the pupils to the nature of fractions, and of the elementary operations upon them, and will throw a great deal of light upon the future progress.

LESSON VII.

The teacher is now to show the class by one or two examples performed by himself upon the blackboard, that sometimes a fraction cannot be multiplied by the second of the operations described above, viz., when the denominator is not divisible by the multiplier. For example, $\frac{3}{5}$ can be doubled by doubling the numerator, making $\frac{6}{5}$, but it cannot be doubled by dividing the denominator, since 5 is not divisible by 2.

QUESTIONS.

Write the fraction $\frac{7}{12}$ upon the slate. Can it be doubled by doubling the numerator? What will be the answer?

Can it be doubled by dividing the denominator? What will be the answer?

Write the fraction $\frac{1}{6}$. Can it be multiplied by three

by multiplying the numerator? What will be the answer?

Can it be multiplied by dividing the denominator? Why not?

Explain also to the class that it is best in general practice to divide the denominator rather than multiply the numerator, where both can be done, as the former gives the answer in a simpler form. Then assign the following examples. Some of them can be performed in both modes and some only in one. When an answer in only one form is to be obtained, the place for the other is to be left blank, and all the work to be arranged in the most neat and workmanlike manner upon the slate, in straight lines and perpendicular columns; thus—

Work upon the Slate.

$$(85.) \quad \frac{1}{3} \bullet 3 = \frac{1^3}{3} = \frac{1}{3}$$

$$(86.) \quad \frac{1}{3} \times 2 = \frac{2}{3}$$

EXAMPLES.

$$85. \quad \frac{1}{3} \times 3$$

$$86. \quad \frac{1}{3} \times 2$$

$$87. \quad \frac{1}{4} \times 2$$

$$88. \quad \frac{2}{7} \times 3$$

$$89. \quad \frac{2}{5} \times 3$$

$$90. \quad \frac{1}{5} \times 3$$

$$91. \quad \frac{1}{5} \times 4$$

$$92. \quad \frac{1}{5} \times 2$$

$$93. \quad \frac{2}{3} \times 9$$

$$94. \quad \frac{2}{3} \times 8$$

$$95. \quad \frac{1}{8} \times 4$$

$$96. \quad \frac{7}{8} \times 5$$

$$97. \quad \frac{7}{8} \times 4$$

$$98. \quad \frac{7}{8} \times 3$$

$$99. \quad \frac{7}{8} \times 2$$

$$100. \quad \frac{1}{1\frac{1}{8}} \times 10$$

$$101. \quad \frac{1}{1\frac{1}{8}} \times 10$$

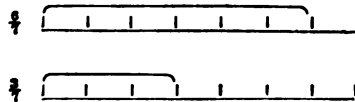
$$102. \quad \frac{2}{1\frac{1}{8}} \times 5$$

$$103. \quad \frac{2}{1\frac{1}{8}} \times 5$$

$$104. \quad \frac{1}{1\frac{1}{8}} \times 4$$

LESSON VIII.

Explain to the class that a fraction may be halved by halving the numerator, leaving the denominator unchanged. Thus $\frac{2}{7}$ halved is obviously one-third, and half of $\frac{2}{7}$ is $\frac{1}{7}$. Represent this also to the eye upon the blackboard, thus—



QUESTIONS.

How much is half of $\frac{1}{2}$? Is it the magnitude or the number of the parts that is diminished? Of $\frac{1}{4}$? Of $\frac{2}{3}$? $\frac{3}{5}$? $\frac{4}{6}$? $\frac{1}{10}$? $\frac{1}{100}$? $\frac{1}{1000}$?

What is half of $\frac{1}{3}$? Of $\frac{2}{3}$? Of $\frac{1}{4}$? Of $\frac{3}{4}$?

Explain to the class that a fraction may be divided by any other number, by dividing the numerator by this number, and render this familiar to them by the following questions.

How much is one third of $\frac{2}{3}$? One quarter of $\frac{1}{2}$? Of $\frac{3}{4}$? Of $\frac{1}{3}$? How much is one-fifth of $\frac{1}{5}$? Of $\frac{2}{5}$? Of $\frac{3}{5}$? How much is one-third of $\frac{2}{3}$? Of $\frac{1}{3}$? Of $\frac{2}{7}$?

Explain to the class also that the sign for division is this, $\div$; and that the expression $\frac{1}{2} \div 2$ means that $\frac{1}{2}$ is to be divided by 2. Then assign the following examples, instructing the class that they are all very easy to be performed, and that the great thing to be attended to is, that the figures and characters are all made in the best forms, that the lines are straight, the columns perpendicular, and not a single error in the whole performance. Then, when they

come to be examined, let all marks of haste and carelessness in the work be pointed out, and those slates, on which the work is most perfectly arranged, be exhibited, so as to awaken a general ambition to produce a workmanlike and handsome style of execution and arrangement.

Work upon the Slate.

$$(105.) \quad \frac{3}{4} \div 2 = \frac{3}{8}$$

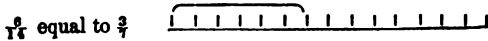
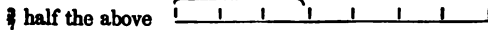
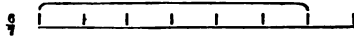
$$(106.) \quad \frac{9}{16} \div 3 = \frac{3}{16}$$

EXAMPLES.

105. $\frac{3}{4} \div 2$	116. $\frac{1}{2} \div 3$
106. $\frac{9}{16} \div 3$	117. $\frac{1}{2} \div 25$
107. $\frac{4}{5} \div 2$	118. $\frac{2}{3} \div 100$
108. $\frac{5}{8} \div 5$	119. $\frac{1}{5} \div 15$
109. $\frac{1}{8} \div 5$	120. $\frac{4}{5} \div 40$
110. $\frac{1}{8} \div 2$	121. $\frac{4}{5} \div 20$
111. $\frac{1}{8} \div 5$	122. $\frac{3}{4} \div 15$
112. $\frac{1}{8} \div 3$	123. $\frac{3}{4} \div 2$
113. $\frac{9}{16} \div 3$	124. $\frac{5}{8} \div 4$
114. $\frac{9}{16} \div 9$	125. $\frac{1}{8} \div 4$
115. $\frac{1}{2} \div 6$	126. $\frac{1}{2} \div 5$

LESSON IX.

Explain to the class that in the examples last performed they have divided the fraction by dividing the *number* of the parts, but that a fraction may be divided by dividing the magnitude of the parts. Thus $\frac{9}{16}$, the first fraction referred to in the last lesson, which then gave $\frac{3}{4}$ when divided by two, may give $\frac{9}{14}$; for, a fourteenth being one-half of a seventh, $\frac{9}{14}$ will be one-half of $\frac{9}{7}$. Illustrate this upon the blackboard in this manner:—



QUESTIONS.

How much is half of $\frac{1}{2}$ if you get the answer by operating on the number of the parts? *Ans.* $\frac{1}{4}$. How much is it if you operate on the magnitude of the parts? *Ans.* $\frac{1}{4}$. Halve $\frac{1}{2}$ by diminishing the number of the parts. Halve $\frac{1}{2}$ by diminishing the magnitude. Halve $\frac{1}{2}$ by making the parts smaller in magnitude. *Ans.* $\frac{1}{4}$. Now halve it by making them smaller in number.

Explain to the class by a divided line upon the blackboard, that, inasmuch as the denomination of the fraction shows how many parts the unit is divided into, the larger the denominator the greater will be the number of these parts, and consequently the smaller each one will be, so that $\frac{1}{10}$ will be but half as great as $\frac{1}{5}$. Thus lead them to observe that multiplying the denominator of a fraction really divides its value, by dividing and rendering smaller the parts. Thus they may either divide the numerator, and make the parts under the vinculum fewer, or they may multiply the denominator, which makes them smaller. Then assign the following examples to be performed by the class in their seats, each one to be performed by both methods, according to the model, the method by multiplying the denominator to be performed first; thus—

Work upon the Slate.

$$(127.) \quad \frac{1}{2} \div 3 = \frac{1}{12} = \frac{1}{6}$$

$$(128.) \quad \frac{1}{2} \div 2 = \frac{1}{4} = \frac{1}{2}$$

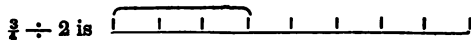
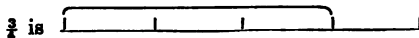
EXAMPLES.

127.	$\frac{2}{3} \div 3$	138.	$\frac{1\frac{2}{3}}{\frac{1}{10}} \div 6$
128.	$\frac{2}{3} \div 2$	139.	$\frac{1\frac{2}{3}}{\frac{1}{10}} \div 5$
129.	$\frac{4}{7} \div 2$	140.	$\frac{2}{3} \div 2$
130.	$\frac{4}{6} \div 2$	141.	$\frac{1\frac{2}{3}}{\frac{1}{10}} \div 4$
131.	$\frac{9}{10} \div 3$	142.	$\frac{1\frac{6}{8}}{\frac{1}{10}} \div 8$
132.	$\frac{1\frac{1}{2}}{\frac{1}{10}} \div 11$	143.	$\frac{9}{10} \div 3$
133.	$\frac{9}{17} \div 4$	144.	$\frac{2\frac{1}{10}}{\frac{1}{10}} \div 7$
134.	$\frac{2\frac{1}{10}}{\frac{1}{10}} \div 7$	145.	$\frac{1\frac{4}{100}}{\frac{1}{100}} \div 7$
135.	$\frac{1\frac{4}{10}}{\frac{1}{10}} \div 7$	146.	$\frac{2}{3} \div 3$
136.	$\frac{2\frac{2}{3}}{\frac{1}{10}} \div 5$	147.	$\frac{1}{6} \div 1$
137.	$\frac{2\frac{2}{3}}{\frac{1}{10}} \div 5$	148.	$\frac{2}{7} \div 3$

LESSON X.

Show now to the class that sometimes when a fraction is to be divided by a given number, the numerator cannot be divided by it, and of course the only mode is to multiply the denominator. For example, if $\frac{3}{4}$ is to be divided by 2, we cannot divide the numerator. The answer, therefore, admits of only one form, which is that obtained by multiplying the denominator, viz., $\frac{3}{8}$.

This should be illustrated also by a line divided thus—



That is, $\frac{3}{4}$ divided by 2 can only be $\frac{3}{8}$; for, since the three-fourths cannot be halved by halving the number of parts, three not being divisible by two, it becomes necessary to halve it by halving the magnitude of the parts; thus leaving the number of the parts still

three, while we make each one-half as great, we change three-fourths to three-eighths, and get half the original value.

The attention of the class must be particularly called to the fact that the larger the denominator the smaller the parts into which the fraction is divided, so that by multiplying the denominator, the numerator remaining the same, the fraction is divided.

QUESTIONS.

Which is the greatest $\frac{1}{2}$ or $\frac{1}{4}$? How much the greatest? Which is greatest $\frac{1}{2}$ or $\frac{1}{4}$? $\frac{3}{4}$ or $\frac{2}{3}$? $\frac{1}{4}$ or $\frac{1}{8}$? How much greater? Which is greatest one $\frac{1}{16}$ or $\frac{1}{8}$? How much greater?

What is double of $\frac{1}{16}$? Of $\frac{1}{8}$? Of $\frac{1}{4}$? Of $\frac{1}{2}$? Of $\frac{1}{4}$?

Can you divide $\frac{3}{4}$ by 2 by diminishing the number of parts? What will be the answer? Can you divide it by 2 by diminishing the magnitude of the parts? What will be the answer?

Assign the following examples to be performed by the class at their seats, the answer to be given in both forms in the cases which admit of it, and in one form in the others, as exemplified in the specimen. The denominator is to be multiplied first, thus :

Work upon the Slate.

$$(149.) \quad \frac{3}{4} \div 3 = \frac{3}{12} = \frac{1}{4}$$

$$(150.) \quad \frac{3}{4} \div 4 = \frac{3}{16}$$

EXAMPLES.

- | | |
|----------------------------|----------------------------|
| 149. $\frac{3}{7} \div 3$ | 160. $\frac{2}{80} \div 2$ |
| 150. $\frac{3}{7} \div 4$ | 161. $\frac{1}{80} \div 5$ |
| 151. $\frac{4}{8} \div 4$ | 162. $\frac{1}{7} \div 10$ |
| 152. $\frac{4}{8} \div 3$ | 163. $\frac{1}{7} \div 10$ |
| 153. $\frac{1}{8} \div 5$ | 164. $\frac{1}{8} \div 3$ |
| 154. $\frac{2}{8} \div 2$ | 165. $\frac{2}{8} \div 5$ |
| 155. $\frac{2}{8} \div 3$ | 166. $\frac{3}{7} \div 3$ |
| 156. $\frac{1}{80} \div 3$ | 167. $\frac{1}{8} \div 2$ |
| 157. $\frac{1}{8} \div 2$ | 168. $\frac{1}{8} \div 5$ |
| 158. $\frac{1}{8} \div 3$ | 169. $\frac{1}{8} \div 15$ |
| 159. $\frac{1}{80} \div 3$ | 170. $\frac{1}{7} \div 7$ |

MISCELLANEOUS EXAMPLES.

171. Four children went to gather blueberries. They were to divide their berries equally when they got home. Mary, one of the party, gave away half of hers. What proportion of the whole number gathered did she have left?

Work upon the Slate.

$$\begin{array}{rcl}
 (171.) & \text{No. of parts} & 4 \\
 & \text{Mary's share} & \frac{1}{2} \\
 & \frac{1}{2} \div 2 & = \frac{1}{4}
 \end{array}$$

172. The next time they went, it was agreed that Mary being older than the rest should have twice as large a share as the rest. What proportion would Mary now have of the whole?

Explain to the class that in this case it would be necessary to make five parts, so as to give two to Mary.

Work upon the Slate.

(172.) For Mary	2 parts.
For the others 1 each	3
Whole number of parts	5
Mary's share	$\frac{2}{5}$

In performing the examples which follow, each pupil is to accompany the work upon the slate with words of explanation like the above, more or less full, according to the discretion of the several pupils. This practice will teach them to think clearly and to express themselves with propriety. They must be instructed also to arrange the work in a neat and symmetrical manner upon the slate. It would be an excellent practice also to provide the class now with copy-books, into which they may transcribe these miscellaneous questions, with the solution of each, the work to be executed and arranged in the books in the most perfect manner. This manuscript should be continued with the miscellaneous examples under the succeeding heads.

173. Four children went out for berries. James and John were to have three shares each, Mary two shares and Sarah one share. When they reached home Mary and Sarah put their shares together; what proportion did they have of the whole?

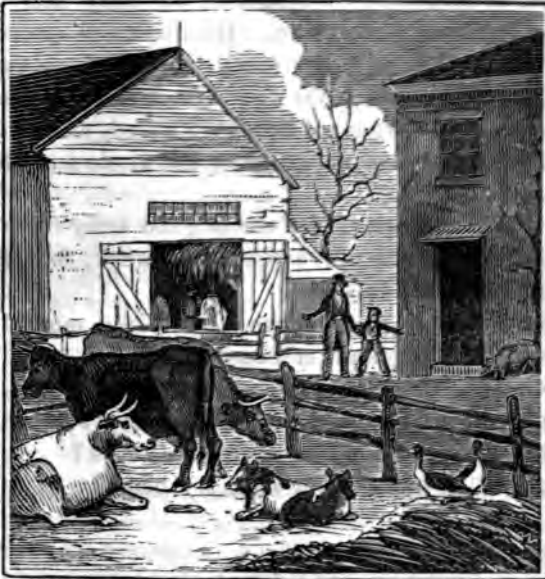
174. James and John sent half of all their share as a present to their aunt; what proportion did she receive?

175. Two boys undertook to measure the fence in front of their father's house. First came a small gate which was 4 ft. wide. Then a piece of fence 15 ft. long. Then a large gate 9 ft. Then a piece of wall eat tree. What proportion of the whole

was the large gate? The small gate? The fence? The wall?

176. A farmer was showing his stock and poultry to his boy. There were three cows worth \$75 each, and three pigs, \$5 each. What proportion of the whole value was in the cows?

177. What proportion was in the pigs?



178. There were two ducks worth together \$1. What proportion was this of the value of all the stock and poultry in the yard?

179. A boy was reading a book containing 316 pages. What proportion of it was the first 120 pages?

180. What fraction would express the remainder?

181. Suppose a man had fifteen dollars to expend in purchases; after he had expended 2 dollars, what portion of the whole would remain? What would remain if he had expended 14 dollars?

182. A boy had a quarter of an apple which he divided into 3 equal portions; how large a part of the whole apple was each of these portions?

183. A general once conducted an army into the field containing 20,000 men. He afterwards received a reinforcement of 5,000 men. He fought several battles. In the first 2,000 were killed, and 100 taken prisoners. In the second 500 were killed, and in the third 1,260. What proportion of his whole force had he now lost?

184. What proportion of the original army?

185. What fraction of a year is one week, considering the year 365 days?

186. What fraction of the year is the month of January? The month of June?

187. What fraction of a week is a day?

188. What fraction of a week is an hour?

189. What fraction of a week is two days?

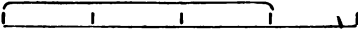
190. What fraction of a week is 6 hours?

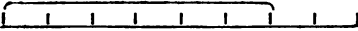
II. CHANGES OF FORM IN FRACTIONAL EXPRESSIONS.

LESSON I.

TO RAISE FRACTIONS TO HIGHER TERMS.

Explain to the pupil that a fraction may be expanded in form, without any alteration of value, by multiplying both numerator and denominator by the same number. Thus $\frac{1}{2}$ becomes $\frac{2}{4}$ when both terms are multiplied by 2. This does not alter the value of the fraction as is seen by the following illustration, in which the vinculum cuts off in the first case $\frac{1}{2}$ and in the second $\frac{2}{4}$.

$\frac{1}{2}$ is 

$\frac{2}{4}$ is 

The principle may also be illustrated by showing the class that by doubling the numerator, we double the number of the parts, but by doubling the denominator we make them all but half as large, so that the value of the expression remains unchanged. Any number used as a multiplier is called a *factor*.

Explain to them also that it sometimes becomes necessary to expand fractions in this way, in order to prepare them to be added more conveniently to other fractions, and that therefore it is necessary for them to learn how to do it. Let them fully understand that expanding a fraction in this way is not *multiplying* it. To multiply a fraction we multiply the numerator *only*—or we *divide*

the denominator *only*—and if we do either of these things we multiply the value. But in raising a fraction to higher terms, in the manner here described, we multiply *both* terms, and the value of the fraction remains unchanged. We multiply the *numerator* and the *denominator*, but we do not multiply the *fraction*.

QUESTIONS.

If we double the denominator of a fraction, do we increase or diminish its value? *Ans.* We diminish it. Why? If we double the numerator, do we increase or diminish the value? Why? If we double both numerator and denominator, what is the effect? *Ans.* The fraction remains the same in value as before.

Work upon the Slate.

$$(1.) \quad \frac{3}{4} = \frac{21}{28} \qquad (2.) \quad \frac{1}{2} = \frac{33}{66}$$

EXAMPLES.

- | | |
|--|--------------------------|
| 1. Expand $\frac{3}{4}$ by the factor 7. | |
| 2. $\frac{1}{2}$ by 8 | 13. $\frac{1}{2}$ by 5 |
| 3. $\frac{2}{3}$ " 5 | 14. $\frac{3}{4}$ " 7 |
| 4. $\frac{3}{4}$ " 2 | 15. $\frac{1}{2}$ " 2 |
| 5. $\frac{2}{3}$ " 5 | 16. $\frac{1}{2}$ " 1 |
| 6. $\frac{1}{2}$ " 3 | 17. $\frac{1}{2}$ " 2 |
| 7. $\frac{1}{2}$ " 2 | 18. $\frac{11}{12}$ " 21 |
| 8. $\frac{1}{2}$ " 5 | 19. $\frac{11}{12}$ " 4 |
| 9. $\frac{7}{8}$ " 2 | 20. $\frac{2}{15}$ " 2 |
| 10. $\frac{11}{12}$ " 12 | 21. $\frac{2}{3}$ " 49 |
| 11. $\frac{2}{3}$ " 6 | 22. $\frac{1}{15}$ " 10 |
| 12. $\frac{1}{2}$ " 2 | 23. $\frac{1}{15}$ " 16 |

LESSON II.

The teacher should here again call the attention of the class to the difference between expanding the form of a fraction as above, and multiplying it. Thus, in expanding $\frac{1}{4}$ by 2, although we multiply both *terms* of the fraction, we do not increase the value of the fraction itself; $\frac{2}{8}$ being the same as $\frac{1}{4}$. If, on the other hand, we wish to multiply the fraction, we must multiply the numerator alone, or divide the denominator alone. Thus, expanding $\frac{2}{3}$ by 2 gives $\frac{4}{3}$, which is the same in value as $\frac{2}{3}$; but multiplying $\frac{2}{3}$ by 2 gives $\frac{4}{3}$, which is twice as great as $\frac{2}{3}$.

EXAMPLES.

- | | |
|---------------------------------|---------------------------------|
| 24. Multiply $\frac{1}{7}$ by 4 | 34. Multiply $\frac{1}{7}$ by 8 |
| 25. " $\frac{2}{8}$ " 2* | 35. " $\frac{1}{8}$ " 9 |
| 26. Expand $\frac{2}{3}$ " 3 | 36. " $\frac{1}{4}$ " 4 |
| 27. " $\frac{4}{5}$ " 7 | 37. Expand $\frac{7}{15}$ " 2 |
| 28. " $\frac{1}{7}$ " 8 | 38. " $\frac{1}{5}$ " 5 |
| 29. Multiply $\frac{2}{3}$ " 3 | 39. Multiply $\frac{4}{5}$ " 5 |
| 30. Expand $\frac{7}{15}$ " 2 | 40. " $\frac{4}{5}$ " 6 |
| 31. " $\frac{2}{15}$ " 9 | 41. " $\frac{2}{5}$ " 3 |
| 32. Multiply $\frac{1}{3}$ " 3 | 42. Expand $\frac{2}{15}$ " 3 |
| 33. Expand $\frac{1}{4}$ " 3 | 43. " $\frac{1}{4}$ " 5 |

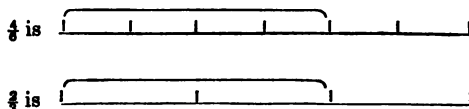
LESSON III.

TO REDUCE FRACTIONS TO LOWER TERMS.

Explain to the pupil that if both the numerator and denominator of a fraction are *divided* by the same number, the value of the frac-

* The pupil should be reminded, that in multiplying a fraction it is best to divide the denominator if possible, otherwise to multiply the numerator.

tion is not altered, but it is reduced to lower terms. The reason of this is, that by dividing the denominator the parts are made larger, and by dividing the numerator the number of them taken is made smaller in the same proportion. This may be illustrated on the blackboard thus. The fraction $\frac{4}{6}$, reduced by dividing both terms by 2, is $\frac{2}{3}$; and these fractions are in value the same.



In the first case the parts are sixths, and in the last case they are thirds, and thirds are twice as large as sixths. Thus, dividing the denominator by 6 makes the parts twice as great. But then there were *four-sixths*, and there are only *two-thirds*. Thus, dividing the numerator makes the number of parts taken but half as great. The one operation, therefore, just balances the other, and the fraction remains the same in value, though it is reduced to lower terms.

Let the pupils fully understand that this dividing both numerator and denominator is not *dividing the fraction*. In order that this may be done, it is necessary for them to see that the fraction itself is a very different thing from the numerator and denominator, used to express it, or from both together. Thus, in the fraction $\frac{2}{3}$, the numerator is *two*, and the denominator is *three*; but the value represented by the expression is not either two or three, but something very different. It is not *five*, which would be two and three together. But it is *two-thirds*, something entirely different from either. This point is so important, and one in regard to which young students are so apt to be confused, that it is worth while for the teacher to take some pains to illustrate it. This may be done if the pupils are quite young, by drawing the figures of two apples upon the blackboard, to show what is the value of the numerator 2; then in a line below *three* apples, to show what is the value of the denominator 3; and then, in a third line, a figure representing *two-thirds*, that is, the portion which would remain if one-

third of the apple were cut away, which would show the value of the fraction $\frac{2}{3}$.

The value of the fraction being thus essentially a different thing from the values of the numerator and denominator expressing it, it does not follow that dividing these terms divides the fraction. On the contrary, if we wish to divide a fraction, it is necessary to divide the numerator and leave the denominator unchanged: or else, to multiply the denominator and leave the numerator unchanged, as has been before explained.

When these points are fully understood by the class, assign the following examples to be performed at their seats.

Work upon the Slate.

$$(44.) \quad \frac{4}{8} = \frac{2}{4}$$

$$(45.) \quad \frac{4}{8} \div 2 = \frac{2}{8}^*$$

EXAMPLES.

44. Reduce $\frac{4}{8}$ by the divisor 2

45. Divide $\frac{4}{8}$ by 2

46. Reduce $\frac{3}{12}$ by the divisor 4

47. Reduce $\frac{7}{14}$ " 7

48. Divide $\frac{7}{14}$ by 7

49. Reduce $\frac{3}{8}$ by the divisor 3

50. " $\frac{4}{12}$ " 4

51. " $\frac{4}{12}$ " 2

* The pupil should be taught, that in dividing a fraction it is best to divide the numerator if possible; otherwise to multiply the denominator.

- | | |
|------------------------------------|-----------------------------------|
| 52. Divide $\frac{7}{16}$ by 7. | 62. Reduce $\frac{11}{12}$ by 2. |
| 53. Multiply $\frac{1}{4}$ by 7. | 63. Expand $\frac{1}{12}$ by 3. |
| 54. Reduce $\frac{11}{12}$ by 3. | 64. Reduce $\frac{3}{4}$ by 3. |
| 55. Reduce $\frac{11}{12}$ by 6. | 65. Multiply $\frac{1}{12}$ by 2. |
| 56. Multiply $\frac{11}{12}$ by 3. | 66. Divide $\frac{1}{12}$ by 2. |
| 57. Divide $\frac{11}{12}$ by 3. | 67. Expand $\frac{1}{3}$ by 100. |
| 58. Multiply $\frac{1}{3}$ by 4. | 68. Reduce $\frac{3}{4}$ by 5. |
| 59. Reduce $\frac{5}{16}$ by 2. | 69. Reduce $\frac{1}{8}$ by 5. |
| 60. Reduce $\frac{1}{8}$ by 3. | 70. Divide $\frac{1}{8}$ by 7. |
| 61. Multiply $\frac{1}{8}$ by 2. | 71. Multiply $\frac{1}{8}$ by 2. |

LESSON IV.

When fractions are to be reduced to their lowest terms they may be divided by any number which will divide both terms. It requires only a little thought and ingenuity to discover what number will divide them when they are small.

QUESTIONS.

Is there any number by which you can divide both terms of this fraction— $\frac{3}{4}$? What number? Is there any number by which you can divide both terms of this fraction— $\frac{3}{7}$? $\frac{4}{5}$? $\frac{9}{16}$? $\frac{3}{8}$? $\frac{3}{5}$? $\frac{1}{4}$? $\frac{2}{4}$? $\frac{3}{16}$? $\frac{5}{16}$? $\frac{3}{7}$? $\frac{3}{8}$? $\frac{3}{5}$? $\frac{3}{8}$? $\frac{3}{5}$? $\frac{3}{8}$?

Reduce all of the following fractions to their lowest terms if they can be reduced. Divide by the largest number which will divide both terms, arranging the work thus :

Work upon the Slate.

$$\frac{3}{4} = \frac{1}{1}.$$

$$(73.) \frac{4}{8} = \frac{1}{2}.$$

EXAMPLES.

72.	$\frac{3}{8}$.	83.	$\frac{3}{11}$.
73.	$\frac{4}{5}$.	84.	$\frac{1}{6}$.
74.	$\frac{3}{6}$.	85.	$\frac{3}{11}$.
75.	$\frac{4}{5}$.	86.	$\frac{17}{14}$.
76.	$\frac{12}{13}$.	87.	$\frac{14}{13}$.
77.	$\frac{12}{13}$.	88.	$\frac{14}{13}$.
78.	$\frac{1}{7}$.	89.	$\frac{14}{13}$.
79.	$\frac{5}{6}$.	90.	$\frac{1}{6}$.
80.	$\frac{5}{6}$.	91.	$\frac{2}{3}$.
81.	$\frac{2}{15}$.	92.	$\frac{2}{3}$.
82.	$\frac{1}{15}$.	93.	$\frac{4}{9}$.

LESSON V.

Explain to the pupil that a number which will exactly divide another number is called a *divisor*. It is also sometimes called a *measure*, because it will exactly measure it. Thus 4 is a divisor of 12, for it will go in it 3 times. In the same manner it may be said to be a *measure* of it, as 4 will measure 12 exactly. But 4 will not measure 13.

If there are two numbers which may be divided by the same divisor, the divisor is said to be *common* to them, because it pertains, as a divisor, equally to both. Thus the numbers 12 and 14 have the common divisor 2, for 2 will divide them both. The phrase *common measure* is sometimes used, which means the same thing.

Have 4 and 8 any common measure? What is it?
 5 and 10? 6 and 7? 4 and 9? 3 and 9? 18 and
 27? 14 and 7? 12 and 13? 11 and 22? 11 and
 2? 2 and 6? 2 and 4? 3 and 6? 5 and 8? 4
 and 8?

Is 3 a common divisor of 7 and 10? Of 9 and 10?
 Of 9 and 12? Of 3 and 5? Of 3 and 6? Of 4 and
 5? Of 4 and 6? Of 5 and 6? Of 9 and 6?

Is 5 a common measure of 9 and 10? Of 15 and
 10? Of 20 and 30?

Explain to the pupils that unless the numerator and denominator of a fraction have a *common* divisor or measure, the fraction cannot be reduced to lower terms; but if they have any common measure at all, the fraction may be reduced by dividing both terms by it. And if the terms have more than one common measure, they may be divided by them in succession, thus, supposing it to be required to reduce $\frac{36}{80}$ to its lowest terms :

Work upon the Slate.

$$(94.) \frac{\overset{2}{36}}{80} = \frac{\overset{3}{12}}{80} = \frac{\overset{5}{6}}{16} = \frac{1}{8}$$

Or the fraction may be reduced at once by dividing by the **greatest** common measure at the outset.

Work upon the Slate.

$$(94.) \frac{\overset{36}{36}}{80} = \frac{1}{8}$$

The answer will always, in the end, be the same, whichever mode is adopted.

EXAMPLES.

Reduce the following fractions to their lowest terms

by dividing first by 2, then by 3, and then by 5, in all cases where these numbers are common measures of both terms.

94.	$\frac{2}{3}\frac{2}{5}$	104.	$\frac{1}{3}\frac{1}{5}$
95.	$\frac{2}{5}\frac{2}{3}$	105.	$\frac{1}{3}\frac{2}{5}$
96.	$\frac{1}{3}\frac{2}{5}$	106.	$\frac{1}{3}\frac{2}{5}$
97.	$\frac{1}{3}\frac{2}{5}$	107.	$\frac{2}{3}\frac{1}{5}$
98.	$\frac{1}{3}\frac{2}{5}$	108.	$\frac{1}{3}\frac{2}{5}$
99.	$\frac{1}{3}\frac{2}{5}$	109.	$\frac{1}{3}\frac{2}{5}$
100.	$\frac{1}{3}\frac{1}{5}$	110.	$\frac{1}{3}\frac{2}{5}$
101.	$\frac{2}{3}\frac{1}{5}$	111.	$\frac{1}{3}\frac{2}{5}$
102.	$\frac{2}{3}\frac{2}{5}$	112.	$\frac{1}{3}\frac{2}{5}$
103.	$\frac{1}{3}\frac{1}{5}$	113.	$\frac{1}{3}\frac{2}{5}$

LESSON VI.

Explain to the class that it is very easy to determine whether any number, however large, is divisible by 2, by observing whether the last figure is odd or even; since if the last figure is even, the whole number is divisible by 2, whereas if the last figure is odd, it is not. A cypher is considered even.

QUESTIONS.

What is the last figure of the number 365? Is it odd or even? Is 365 then divisible by 2? What is the last figure of 788? Odd or even? Is 2 a measure of 788? Is 2 a measure of 789? Of 700? Of 1,828? Of 78,964? Of 78,966?

It is also easy to determine whether any number is divisible by 3, by observing whether all the figures which compose it, added together make a sum which is divided by 3. Thus 111 consists of 3*

three ones, which added together make 3,—therefore the number 111 is divisible by 3. It will interest the class and impress the fact upon their minds to divide some such numbers upon the black-board.

In the same manner 201, 102, &c., will be divisible by 3. Also 4,200, 12,111, because the figures, all added together, make 6, which is divisible by 3. So 90,111, 7,020,111, because the figures, all added together, make numbers divisible by 3.

QUESTIONS.

Is 45 divisible by 3? 120? 10,101? 34? 304? 30,004? 30,003? Is 405 divisible by 2? by 3? Is 1,002 divisible by 3? by 2?

It is also easy to determine whether a number is divisible by 5, by observing the last figure. If the last figure is 5 or 0, then the whole number is divisible by 5. Otherwise not.

Is 5 a measure of 350? Is 3? Is 2? Is 5 a measure of 795? Is 2? Is 5 a measure of 10,101? Is 3? Is 2?

Reduce the following fractions to lower terms by dividing first by 2, as long as the last figure of each continues even; then by 3 as long as the figures of both terms added together make an even number of threes, and then by 5 as long as the last figure of each term continues 5 or 0.

Work upon the Slate.

$$(114.) \quad \frac{\overset{2}{\cancel{36220}}}{\cancel{11655}} = \frac{\overset{5}{\cancel{18460}}}{\cancel{23311}} = \frac{\cancel{3682}}{\cancel{4662}}$$

$$(115.) \quad \frac{\overset{2}{\cancel{240}}}{\cancel{120}} = \frac{\overset{3}{\cancel{120}}}{\cancel{120}} = \frac{\overset{2}{\cancel{60}}}{\cancel{60}} = \frac{\overset{3}{\cancel{30}}}{\cancel{30}} = \frac{\overset{5}{\cancel{15}}}{\cancel{15}} = \frac{1}{1}$$

EXAMPLES.

114.	$\frac{3}{4}\frac{4}{11}\frac{2}{3}\frac{2}{5}$	125.	$\frac{1}{2}\frac{2}{3}$
115.	$\frac{2}{7}\frac{4}{3}\frac{2}{5}$	126.	$\frac{1}{2}\frac{1}{3}$
116.	$\frac{4}{7}\frac{2}{3}\frac{2}{5}$	127.	$\frac{1}{2}\frac{2}{3}$
117.	$\frac{1}{7}\frac{2}{3}\frac{2}{5}$	128.	$\frac{1}{2}\frac{2}{3}$
118.	$\frac{2}{11}\frac{4}{3}\frac{1}{4}\frac{2}{5}\frac{2}{3}$	129.	$\frac{7}{11}\frac{2}{3}\frac{2}{5}$
119.	$\frac{1}{11}\frac{2}{3}\frac{2}{5}$	130.	$\frac{4}{3}\frac{2}{5}$
120.	$\frac{2}{3}\frac{1}{11}\frac{2}{5}\frac{2}{3}$	131.	$\frac{2}{3}\frac{2}{5}$
121.	$\frac{1}{3}\frac{2}{5}$	132.	$\frac{7}{3}\frac{2}{5}\frac{2}{3}$
122.	$\frac{2}{3}\frac{2}{5}\frac{2}{3}$	133.	$\frac{1}{3}\frac{2}{5}\frac{2}{3}$
123.	$\frac{1}{3}\frac{1}{11}\frac{2}{5}$	134.	$\frac{1}{3}\frac{2}{5}\frac{2}{3}\frac{2}{5}$
124.	$\frac{1}{11}\frac{1}{3}\frac{1}{5}$	135.	$\frac{2}{3}\frac{1}{5}$

LESSON VII.

Explain now to the class that though it is easy to determine whether any number can be measured or divided by 2, 3, or 5, it is much more difficult in the case of other measures. For instance, take the fraction $\frac{2}{3}\frac{2}{7}\frac{2}{5}$. The terms cannot both be divided by either 2, 3, or 5, but there is a number by which they can both be divided, though it would be very difficult for an inexperienced person to discover what it is. The class can try to see if they can discover it, and if they should fail, the teacher may tell them it is 13.

The teacher may then explain, that mathematicians have discovered a mode by which the greatest common divisor of any two numbers may always be found, which is this. Divide the greatest of the two numbers by the least, and then bring down the least, and divide it by the remainder left after the first division. This is to be continued until there is no remainder. Then the number last divided by, that is, the one that left no remainder, will divide both the original numbers, and it will be the greatest number which will divide them. That is, it is the *greatest common divisor*, or in other words, the *greatest common measure*.

The teacher should then perform the operation thus ; supposing it were required to find the greatest number which will divide both 105 and 385.

$$\begin{array}{r}
 105 \overline{) 385} \text{ (3} \\
 \underline{315} \\
 70 \overline{) 105} \text{ (1} \\
 \underline{70} \\
 \text{Ans. } 35 \overline{) 70} \text{ (2} \\
 \underline{70} \\
 0
 \end{array}$$

It will be necessary for the teacher to go over this process a great many times in the class, before they will become familiar with it. He may perform the above example upon the blackboard, in their presence, explaining carefully every step. Then he may give the numbers to the class, requesting them to try to perform the operation upon their slates, looking up at the blackboard for guidance, if they find it necessary, but not otherwise. Some will perhaps not be able to perform the work at all. Others will succeed in doing it by the help of the work before them on the board. Others perhaps may perform it without this aid. The teacher should be satisfied with the effort, and not take any particular notice of those who fail, but encourage all by saying it is not surprising that they find it difficult, since this is one of the processes which most frequently perplex beginners, and that after he explains it once or twice more they will all understand it. Then let him take two new numbers, as 385 and 1365, which will give for their greatest common measure 35. The steps should be explained slowly and carefully, and then the class be allowed to attempt the work themselves. In the same manner, a sufficient number of the examples given below may be performed in the class, to make all acquainted with the process. It is better that they should not commit to memory any rule, as in such an attempt their minds are mainly occupied with the *sounds of the words which they are to re-*

peal, instead of being directed to *the real nature of the process*, as a series of operations which they are to *perform*.

It is desirable also here to remind the pupils that the only reason why we want to find the greatest common measure of any two numbers, is so as to know what number to divide the numerator and denominator of a fraction by, in order to reduce it to its lowest terms.

EXAMPLES.

136. Find the greatest common measure of 102 and 595.

137. Suppose you wish to reduce $\frac{1}{4}$ to its lowest terms, what would be the greatest common divisor?

138. What is the greatest common divisor of 64 and 24?

139. Find the greatest common measure of 80 and 90.

140. Of 920 and 360

148. Of 64 and 620

141. Of 240 and 24

149. Of 720 and 620

142. Of 360 and 240

150. Of 132 and 66

143. Of 78 and 56

151. Of 500 and 1200

144. Of 94 and 108

152. Of 144 and 180

145. Of 99 and 108

153. Of 96 and 800

146. Of 801 and 108

154. Of 16 and 56

147. Of 900 and 600

155. Of 14 and 140

LESSON VIII.

After examining the results obtained in performing the last examples, the teacher should again remind the pupils of the true nature of a greatest common measure, and show them that they can often tell at once, by inspection, what the greatest common measure is, without making the calculation described in the last lesson; though that calculation will always give it, when made. If one of the numbers will divide the other, then that one must itself be the greatest common measure of the two. Thus, suppose

numbers 14 and 140 are given, and it is required to find their greatest common measure. Now, as 14 will measure 140, being contained in it just ten times, and as it will of course measure itself, it will be the greatest common measure of the two. In such cases as this, therefore, it is not necessary to go through the process above described, as the greatest common measure can readily be seen.

Explain also to the pupils that the only occasion they will have to find the greatest common measure of two numbers is in case of a fraction which they wish to reduce to lower terms. That when such a fraction is given, they are to find first the greatest common measure of the numerator and denominator, considered as two independent numbers. If they can see at once what this measure is, it is unnecessary to go through the process of calculation. If they cannot determine it by inspection, they can calculate it as explained in the last lesson. When the greatest common measure is found, the numerator and denominator of the fraction are both to be divided by it, which will give a new numerator and denominator, by which the value of the fraction is expressed in its lowest terms. The pupil must here be reminded that it is the last *divisor*, and not the quotient, which is the greatest common measure.

156. Reduce $\frac{3}{4}\frac{3}{4}\frac{3}{4}$ to its lowest terms.

Work upon the Slate.

$$\begin{array}{r}
 (156.) \quad 280)346(1 \\
 \underline{280} \\
 66)280(4 \\
 \underline{264} \\
 16)66(4 \\
 \underline{64}
 \end{array}$$

Greatest common measure 2)16(8
16

$$\text{Then } \frac{\frac{3}{4}\frac{3}{4}\frac{3}{4}}{\frac{2}{2}\frac{2}{2}\frac{2}{2}} = \frac{3}{4}\frac{3}{4}\frac{3}{4} \text{ Ans.}$$

EXAMPLES.

Reduce the following fractions to their lowest terms :

157.	$\frac{340}{170}$	167.	$\frac{100}{100}$
158.	$\frac{120}{120}$	168.	$\frac{1600}{1600}$
159.	$\frac{720}{720}$	169.	$\frac{342}{171}$
160.	$\frac{600}{600}$	170.	$\frac{1}{1}$
161.	$\frac{22}{11}$	171.	$\frac{22}{11}$
162.	$\frac{14}{7}$	172.	$\frac{18}{9}$
163.	$\frac{170}{170}$	173.	$\frac{16}{8}$
164.	$\frac{314}{314}$	174.	$\frac{21}{21}$
165.	$\frac{140}{140}$	175.	$\frac{1}{1}$
166.	$\frac{160}{160}$	176.	$\frac{6}{6}$

LESSON IX.

In some cases numbers have no common measure greater than 1. They are then said to be *incommensurable*. To illustrate this, the teacher may take two incommensurable numbers, as 135 and 622, and dividing one by the other, and the last divisor by the remainder, as explained in Lesson 7, and it will be found that no common measure greater than 1 will result.

The pupil should also be taught that when the numerator and denominator are *incommensurable*, the fraction cannot be reduced to lower terms. Then assign the following fractions to be reduced to their lowest terms in all cases where they are capable of such reduction.

EXAMPLES.

177.	$\frac{277}{277}$	179.	$\frac{11}{11}$
178.	$\frac{277}{277}$	180.	$\frac{11}{11}$

181.	$\frac{2}{3}\frac{2}{3}\frac{2}{3}$	189.	$\frac{1}{3}\frac{2}{3}$
182.	$\frac{3}{4}\frac{1}{2}$	190.	$\frac{1}{3}\frac{2}{3}$
183.	$\frac{1}{2}\frac{2}{3}$	191.	$\frac{1}{3}\frac{2}{3}$
184.	$\frac{1}{2}\frac{1}{2}$	192.	$\frac{1}{2}\frac{2}{3}$
185.	$\frac{1}{2}\frac{2}{3}$	193.	$\frac{1}{2}$
186.	$\frac{1}{3}\frac{4}{6}$	194.	$\frac{2}{3}$
187.	$\frac{1}{2}\frac{2}{3}$	195.	$\frac{1}{2}\frac{1}{2}$
188.	$\frac{1}{2}\frac{2}{3}$	196.	$\frac{2}{3}$

LESSON X.

Explain to the pupil that it is immaterial whether the terms of a fraction are divided by the greatest common measure once for all, or by the smaller common measures in succession. For example, if the fraction $\frac{120}{180}$ were given, we might reduce it in either of the following ways:

1. We may divide it by the greatest common measure at once; thus:

$$\begin{array}{r} 120) 180 (1 \\ \underline{120} \\ \text{Greatest com. meas. } 60) 120 (2 \\ \underline{120} \end{array}$$

$$\text{Then } \frac{120}{180} = \frac{2}{3} \text{ Ans.}$$

Or 2. We may divide it by the smaller common measures in succession, thus:

$$\frac{120}{180} = \frac{2}{3}\frac{2}{3} = \frac{2}{3}\frac{2}{3} = \frac{2}{3}\frac{2}{3} = \frac{2}{3} \text{ Ans. same as before.}$$

It will interest the class to observe, that in all cases like this the smaller common measures multiplied together will always make a measure. Thus, in the case above given, the

smaller measures multiplied together $2 \times 2 \times 3 \times 5 = 60$, which is the greatest common measure.

In practice, it is convenient to divide the terms of a given fraction by the measures 2, 3 and 5, as far as possible, and then to get the greatest common measure (if there is any) of the terms which result. Thus, in order to reduce $\frac{1340}{4400}$ to the lowest terms, we first divide successively by the small measures.

$$\frac{1340}{4400} = \frac{670}{2200} = \frac{335}{1100} = \frac{67}{220}$$

Then obtain the greatest common measure of 45 and 163, if there is any.

$$\begin{array}{r} \text{Then } 45 \overline{)163} 3 \\ \underline{135} \\ 28 \\ 28 \overline{)45} 1 \\ \underline{28} \\ 17 \\ 17 \overline{)28} 1 \\ \underline{17} \\ 11 \\ 11 \overline{)17} 1 \\ \underline{11} \\ 6 \\ 6 \overline{)11} 1 \\ \underline{6} \\ 5 \\ 5 \overline{)6} 1 \\ \underline{5} \\ 1 \\ 1 \overline{)5} 5 \\ \underline{5} \end{array}$$

The result shows that 45 and 163 are *incommensurable*, and that consequently $\frac{67}{220}$ is in its lowest terms.

On the other hand, in the case of $\frac{10140}{11110}$, we reduce first,

$$\overset{2}{\cancel{1111}} = \overset{3}{\cancel{111}} = \overset{5}{\cancel{111}} = \cancel{111}$$

Then, by the process above,

$$\begin{array}{r} 338)507(1 \\ \underline{338} \\ 169)338(2 \\ \underline{338} \end{array}$$

We find that 169 is the greatest common measure of 338 and 507, and continuing the reduction we have

$$\overset{100}{\cancel{111}} = \frac{1}{3} \text{ Ans.}$$

So that it appears that $\frac{100}{111}$ is equal to exactly $\frac{1}{3}$.

Ordinary pupils are not to be expected to understand this from the description here. The teacher must explain it to them orally, and illustrate it repeatedly by examples, performed by himself upon the blackboard, and repeated by them upon their slates, until all are familiar with the process. The following examples may then be assigned to be performed by the class, at their seats, in the foregoing manner, viz., by dividing both the terms by the measures 2, 3 and 5, as long as possible, and finding the greatest common measure, if any, of the results, and then finishing the reduction by dividing by that common measure.

EXAMPLES.

197.	$\frac{370}{1110}$	204.	$\frac{2}{3}$
198.	$\frac{840}{1110}$	205.	$\frac{1}{3}$
199.	$\frac{111}{111}$	206.	$\frac{1}{11}$
200.	$\frac{3}{1}$	207.	$\frac{1}{3}$
201.	$\frac{3}{1}$	208.	$\frac{1}{3}$
202.	$\frac{110}{1110}$	209.	$\frac{11}{111}$
	$\frac{1}{111}$	210.	$\frac{11}{111}$

211.	$\frac{4}{3}\frac{2}{3}$	214.	$\frac{1}{3}\frac{1}{3}$
212.	$\frac{1}{3}\frac{1}{3}$	215.	$\frac{4}{3}\frac{2}{3}$
213.	$\frac{4}{3}\frac{2}{3}$	216.	$\frac{1}{3}\frac{1}{3}$

LESSON XI.

TO CHANGE A WHOLE NUMBER TO THE FORM OF A FRACTION.

The teacher may draw two lines upon the blackboard to represent the number 2, thus :—

2. 

Then let him divide each line into three parts, and show the class that as there are three thirds in each line, there will be six thirds in two. The value of the whole number 2 may therefore be expressed by the fraction $\frac{6}{3}$, thus :—

$\frac{6}{3}$ 

QUESTIONS.

How many thirds are there in 3? What fraction then will represent the whole number 3? How many thirds in 4? What fraction will represent it? In 5? In 6? In 7? In 8? In 9? In 10? In 1?

How many fourths are there in 2? What fraction then will represent two in fourths? 3? 4? 5? 6? 7? 8? 9? 10? 1?

How many fifths in two? What fraction will represent 2 in fifths? 3? 4? 5? 6? 7? 8? 9? 10? 1?

How many sixths in two? What fraction will represent 2 in sixths? 3? 4? 5? 6? 7? 8? 9? 10? 1?

Show to the pupil that in order to reduce a whole number to the form of a fraction, if the whole number be 3 and we want to reduce it to sixths, it is clear that it must make three times six, or *eighteen* sixths, for there are six sixths in one, and of course there must be eighteen sixths in three. Then put questions in the following form :

How many ninths are there in one ? Then how many will there be in three ? How many thirds in one ? How many in five ? How many tenths in one ? How many in four ? How many hundredths in one ? How many in five ?

Reduce 4 to a fraction whose denominator shall be 3 ; that is, reduce it to thirds. Reduce 2 to a fraction whose denominator shall be 3. Reduce 2 to a fraction, denominator 5. Reduce 5 to a fraction, denominator 4.

Continue such questions until the pupils are all familiar with the operation. Be careful, in doing this, not to hurry them, but remember, that though it seems very simple to us, it is very abstruse, and may be very perplexing, to them. If any one is at a loss, he is to be aided immediately, unless it is obvious that his hesitation arises from carelessness and inattention ; and even in those cases, kind words and encouragement will usually be far more effectual in awakening the dormant interest and attention, than reproofs or other indications of displeasure. After a short time, all the class will see that in these cases, where a whole number is given to be reduced to a fraction of a particular denomination, we multiply the whole number by the proposed denominator ; this will give the numerator of the fraction, and this numerator written over the proposed denominator, will be the fraction required. Tell them that this is what is usually called the rule, but that the great thing is for them to understand and remember *the process itself*, by means of a distinct understanding of the nature and ground of it, rather than to rely on a certain form of words describing it.

Then let a few of the following examples be performed by the class together, so as to be sure that all understand the method of operation. When this is ascertained the remainder may be assigned to be performed by the pupils at their seats. The work should be arranged thus:—

217. Reduce 41 to a fraction whose denominator shall be 9.

Work upon the Slate.

217. 41 whole number.
9 proposed denominator.

369 numerator.

$41 = \frac{369}{9} =$ fraction required.

EXAMPLES.

218. Reduce 468 to thirds.

219. Reduce 321 to a fraction ; denominator to be 15.

220. How many sixteenths in 281 ?

221. Reduce 490 to a fraction ; denominator 48.

222. " 452 " " 17.

223. " 95 " " 5.

224. " 2122 " " 4.

225. " 2146 " " 6752.

226. " 349 " " 254.

227. " 111 " " 2.

228. " 700 " " 10.

229. " 50 " " 5.

230. " 750 " " 16.

231. " 3 " " 2.

232. " 560 " "

233. " 99 " "

LESSON XII.

TO REDUCE A MIXED NUMBER TO THE FORM OF A FRACTION.

After examining the work performed by the pupils at their seats, under the last lesson, call their attention to the fact that in all the answers the numerators are greater than the denominators, and that the value of the fraction is greater than 1. In most cases of fractions, as they occur in common use, the value is less than 1. Thus $\frac{3}{4}$ is only a part of an apple. But $\frac{5}{4}$ of an apple is equal to two apples, and therefore can hardly be said to be properly a *fraction*, which means a *portion* separated from the rest. Still, as such expressions as $\frac{5}{4}$, $\frac{7}{4}$, &c., have both a numerator and a denominator, and as they are in all respects in the form of fractions, they are called by that name; but to distinguish them from the others, they are called *improper fractions*.

Explain also that sometimes an expression consists of a whole number and a fraction together, as $4\frac{2}{3}$, which is read *four and two-thirds*. This is called a *mixed number*, being partly a whole number and partly a fraction. To convert $4\frac{2}{3}$ into thirds, we first change the 4 into thirds, as in the former lesson, and then add the two thirds. Thus, there are 12 thirds in 4, and the 2 besides make $\frac{14}{3}$. Explain this fully, and with several illustrations upon the blackboard, and then propose the following questions:—

QUESTIONS.

How many thirds are there in 1? In $1\frac{1}{3}$? $1\frac{2}{3}$?
2? $2\frac{1}{3}$? $2\frac{2}{3}$? 3? $3\frac{1}{3}$? $3\frac{2}{3}$? 4?

How many fourths are there in 2? In $2\frac{1}{4}$? $3\frac{1}{4}$?
 $4\frac{1}{4}$? $4\frac{2}{4}$? $4\frac{3}{4}$?

Continue to ask these and similar questions until the philosophy of the process is well understood, and then assign the following mixed numbers to be reduced to improper fractions.

EXAMPLES.

234.	$5\frac{2}{7}$	244.	$14\frac{4}{15}$
235.	$21\frac{2}{3}$	245.	$19\frac{1}{2}$
236.	$456\frac{2}{4}$	246.	$160\frac{7}{15}$
237.	$159\frac{1}{4}$	247.	$350\frac{1}{4}$
238.	$33\frac{1\frac{4}{5}}{10}$	248.	$19\frac{1}{15}$
239.	$24\frac{1\frac{2}{3}}{4}$	249.	$19\frac{2}{15}$
240.	$18\frac{2}{15}$	250.	$270\frac{2}{17}$
241.	$71\frac{2}{3}$	251.	$220\frac{1}{4}$
242.	$18\frac{1}{4}$	252.	$44\frac{7}{15}$
243.	$17\frac{1}{2}$	253.	$12\frac{1}{15}$

LESSON XIII.

TO REDUCE AN IMPROPER FRACTION TO A WHOLE OR MIXED NUMBER.

Explain to the pupils that this process is only the reverse of the preceding, as they will perceive by answering the following questions :—

QUESTIONS.

How many units or wholes are there in $\frac{4}{3}$? How many in $\frac{7}{4}$? *Ans.* Two and one-third. How many in $\frac{2}{3}$? In $\frac{1}{3}$? In $\frac{2}{4}$? In $\frac{1}{4}$? In $\frac{2}{5}$? In $\frac{1}{5}$?

Call the attention of the pupils to the fact that in such cases as these the mental process by which they obtain the result is a division of the numerator by the denominator, which gives the whole number, and that part of the numerator which remains written over the denominator, is the fractional part of the answer.

Illustrate this practically upon the blackboard by taking numbers too large to be easily managed in the mind. Thus, to reduce $\frac{613}{220}$ to a mixed number, we say—

220) 613(2 whole number.

440

173 portion of the numerator remaining.

We thus get 2 for the whole number, and $\frac{173}{220}$ for the fraction. In order that this process may be clearly understood, the class should be questioned upon the operation thus: What is the denomination of the parts in the fraction? *Ans.* Two-hundred-and-twentieths. How many of these parts are there expressed in the fraction? *Ans.* 613. How many such parts will it take to make 1? *Ans.* 220. Are there enough then to make one whole? How many will it take to make 2 wholes? *Ans.* 440. [Here the 440 should be pointed out in the above operation as obtained by multiplying 220 by 2.] And how many will remain? *Ans.* 173. Here point out the remainder, 173. This is 173 what? *Ans.* 173 two-hundred-and-twentieths. The process should be repeated with other examples, and similar questions be asked in respect to them, until both the order of the steps and the nature and philosophy of them are perfectly familiar.

QUESTIONS.

How many wholes are there in $\frac{7}{3}$? How many of the $\frac{7}{3}$ are required to make the 2 wholes? How many of the seven-thirds are remaining? Then what will $\frac{7}{3}$ become, reduced to a mixed number?

How many wholes in $\frac{23}{10}$? How many tenths will it take for the 2 wholes? How many tenths remain? Then what will $\frac{23}{10}$ become, expressed as a mixed number?

How many wholes in $\frac{3}{10}$? How many of the 30 tenths does it take to make 3? How many then remain? What then does $\frac{3}{10}$ become when reduced?

It appears by the last case that sometimes the improper fraction is reducible to a whole number complete, with no fraction remaining. This is called reducing the fraction to a whole number. On the other hand, when there is a fraction in the answer, it is to be joined to the whole number. It often cannot be determined beforehand, what will be the form of the answer; as, for instance, in the case of the fraction $\frac{85}{17}$, one could not at first determine whether 85 seventeenths would make an exact number of units or not; but on dividing it we find there are just 5 wholes in 85 seventeenths.

EXAMPLES.

254. Reduce $\frac{4}{3}$ to a whole or mixed number.

255.	$\frac{3^3 3}{3^0}$	265.	$\frac{4^4 0}{1^0}$
256.	$\frac{1^3}{1^0}$	266.	$\frac{2^2 0}{1^0}$
257.	$\frac{1^4 0}{1^0}$	267.	$\frac{1^3}{1^0}$
258.	$\frac{1^7 0}{1^0}$	268.	$\frac{5^0 1}{1^0}$
259.	$\frac{2^1}{1^0}$	269.	$\frac{2^3 0}{1^0}$
260.	$\frac{3^3 0}{1^0}$	270.	$\frac{1^3}{1^0}$
261.	$\frac{1^1 1^1 1^1}{1^0}$	271.	$\frac{2}{1^0}$
262.	$\frac{2^2 0}{1^0}$	272.	$\frac{4}{1^0}$
263.	$\frac{1^1 0}{1^0}$	273.	$\frac{1^3 3}{1^0}$
264.	$\frac{0^2 4}{1^0}$	274.	$\frac{0^7 0}{1^0}$

LESSON XIV.

TO REDUCE COMPOUND FRACTIONS TO SIMPLE ONES.

A compound fraction is one of this form, $\frac{1}{2}$ of $\frac{3}{4}$. Show the pupils that it is easy to ascertain what the value of such an expression

is, in the above case, where the numerator of the first fraction is 1. For we have only to get one-quarter of $\frac{1}{2}$. But to get a quarter of any quantity we have only to divide it by 4. And as has already been shown, we get a quarter of any fraction by dividing the numerator or multiplying the denominator. We cannot divide the numerator, so we multiply the denominator; $\frac{1}{4}$ of $\frac{1}{2}$ would therefore be $\frac{1}{8}$.

QUESTIONS.

What is $\frac{1}{4}$ of $\frac{2}{3}$? $\frac{1}{4}$ of $\frac{2}{4}$? $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{2}{3}$?

When this is understood, the following examples, having numbers often too large to be multiplied mentally, may be assigned to be performed upon the slate.

EXAMPLES.

275.	$\frac{1}{12}$	of	$\frac{2}{3}$	285.	$\frac{1}{2}$	of	$\frac{12}{357}$
276.	$\frac{1}{4}$	of	$\frac{2}{35}$	286.	$\frac{1}{18}$	of	$\frac{1}{25}$
277.	$\frac{1}{2}$	of	$\frac{1}{12}$	287.	$\frac{1}{12}$	of	$\frac{1}{15}$
278.	$\frac{1}{18}$	of	$\frac{1}{325}$	288.	$\frac{1}{18}$	of	$\frac{1}{22}$
279.	$\frac{1}{18}$	of	$\frac{1}{22}$	289.	$\frac{1}{18}$	of	$\frac{1}{65}$
280.	$\frac{1}{2}$	of	$\frac{1}{18}$	290.	$\frac{1}{2}$	of	$\frac{1}{15}$
281.	$\frac{1}{2}$	of	$\frac{1}{12}$	291.	$\frac{1}{2}$	of	$\frac{1}{25}$
282.	$\frac{1}{2}$	of	$\frac{1}{18}$	292.	$\frac{1}{12}$	of	$\frac{1}{15}$
283.	$\frac{1}{12}$	of	$\frac{1}{1855}$	293.	$\frac{1}{12}$	of	$\frac{1}{1555}$
284.	$\frac{1}{2}$	of	$\frac{1}{18}$	294.	$\frac{1}{2}$	of	$\frac{1}{18}$

LESSON XV.

Call the attention of the class to the fact that in all the preceding ; denominator of the first fraction is 1, which simplifies

the process in these cases. But in case the numerator is more than 1, we have another step to take, time :-

If we wish to get $\frac{1}{4}$ of $\frac{1}{2}$, we have the answer as above, $\frac{1}{12}$, which is obtained by multiplying the denominators together. Now if we wish to get $\frac{3}{4}$ of $\frac{1}{2}$, it is evident we have only to take three times as much, which will be $\frac{1}{4}$.

Therefore, to get one-quarter of $\frac{1}{2}$, we multiply the denominator by 4; and to get three-quarters we then multiply the numerator by 3.

QUESTIONS.

What is $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{3}{4}$ of $\frac{1}{2}$? $\frac{1}{8}$ of $\frac{1}{2}$? $\frac{3}{8}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{3}{4}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{1\frac{1}{2}}$? $\frac{3}{4}$ of $\frac{1}{1\frac{1}{2}}$?

Although it is desirable that the pupils should understand the philosophy of such a process, it is not necessary that they should always have it in mind, in performing the work. When, therefore, the teacher finds that the class comprehend fully the nature of the two steps, he may show them that the work becomes merely the multiplying of the numerators together for a new numerator, and the denominators for a new denominator, and that, without stopping to consider the reason of this, every time they execute the work, all they have to do is to perform the multiplications as quick as they can. To accustom them to this rapidity of execution, he may write small compound fractions upon the blackboard, and let the class see how quick they can reduce them. They will soon learn to reduce them faster than the teacher can write them. Then let the following examples, which consist of numbers so large, in most cases, as to require multiplication upon the slate, be assigned to be performed at the seats. The answers are to be given without being reduced to lower terms.

EXAMPLES.

$$295. \quad \frac{1}{4} \text{ of } \frac{1}{1\frac{1}{2}}$$

$$296. \quad \frac{3\frac{1}{2}}{1\frac{1}{2}} \text{ of } \frac{1}{1\frac{1}{2}}$$

$$297. \quad \frac{1}{1\frac{1}{2}} \text{ of } \frac{1}{1\frac{1}{2}}$$

$$298. \quad \frac{1}{1\frac{1}{2}} \text{ of } \frac{1}{1\frac{1}{2}}$$

299.	$\frac{1}{12}$	of	$\frac{1}{15}$	307.	$\frac{3}{7}$	of	$\frac{3}{15}$
300.	$\frac{3}{8}$	of	$\frac{7}{8}$	308.	$\frac{1}{8}$	of	$\frac{1}{8}$
301.	$\frac{1}{10}$	of	$\frac{3}{2} \frac{5}{11}$	309.	$\frac{7}{10}$	of	$\frac{1}{7}$
302.	$\frac{1}{8}$	of	$\frac{1}{8}$	310.	$\frac{2}{10}$	of	$\frac{7}{10}$
303.	$\frac{3}{14}$	of	$\frac{2}{3}$	311.	$\frac{1}{12}$	of	$\frac{7}{12}$
304.	$\frac{2}{10}$	of	$\frac{1}{8}$	312.	$\frac{1}{8}$	of	$\frac{1}{8}$
305.	$\frac{1}{2}$	of	$\frac{1}{12}$	313.	$\frac{1}{2}$	of	$\frac{1}{2}$
306.	$\frac{1}{12}$	of	$\frac{1}{12}$	314.	$\frac{1}{2}$	of	$\frac{1}{2}$

LESSON XVI.

When a compound fraction has been reduced to a single one, as in the examples contained in the last two lessons, it will often happen that the result at first obtained may be reduced to lower terms. For example, if we have the expression $\frac{2}{3}$ of $\frac{9}{16}$ to be reduced to a simple fraction, by multiplying the numerators together, and also the denominators, as before explained, we get $\frac{18}{48}$. Now $\frac{18}{48}$ may be reduced to lower terms, thus :—

$$\frac{\overset{3}{18}}{\underset{3}{48}} = \frac{\overset{3}{9}}{\underset{3}{16}} = \frac{3}{16} \text{ so that } \frac{1}{16} \text{ is really } \frac{3}{16}$$

Consequently $\frac{2}{3}$ of $\frac{9}{16}$ is $\frac{3}{8}$.

Let the teacher explain this fully to the class, illustrating it upon the blackboard, and then give them several examples of a similar kind to be performed in the class, until he is sure that all the pupils understand the process. When this is done, the following examples may be assigned to be performed by each one at his seat :—

EXAMPLES.

315.	$\frac{1}{2}$	of	$\frac{1}{2}$	318.	$\frac{2}{3}$	of	$\frac{1}{12}$
316.	$\frac{3}{4}$	of	$\frac{2}{3}$	319.	$\frac{1}{2}$	of	$\frac{1}{10}$
317.	$\frac{1}{2}$	of	$\frac{1}{3}$	320.	$\frac{1}{2}$	of	$\frac{1}{2}$

321.	$\frac{1}{3}$	of	$\frac{3}{4}$	328.	$\frac{1}{2}$	of	$\frac{3}{4}$
322.	$\frac{1}{2}$	of	$\frac{1}{5}$	329.	$\frac{1}{2}$	of	$\frac{2}{3}$
323.	$\frac{1}{4}$	of	$\frac{3}{8}$	330.	$\frac{1}{8}$	of	$\frac{3}{8}$
324.	$\frac{3}{4}$	of	$\frac{1}{2}$	331.	$\frac{1}{8}$	of	$\frac{1}{8}$
325.	$\frac{1}{3}$	of	$\frac{3}{4}$	332.	$\frac{1}{4}$	of	$\frac{1}{8}$
326.	$\frac{1}{2}$	of	$\frac{1}{3}$	333.	$\frac{3}{4}$	of	$\frac{1}{8}$
327.	$\frac{1}{8}$	of	$\frac{2}{7}$	334.	$\frac{3}{4}$	of	$\frac{3}{4}$

MISCELLANEOUS EXAMPLES.

1. A gentleman taking a journey paid \$5 for rail road fare, \$10 on board a steamboat, and \$9 at a hotel. He had \$36 when he set out. What proportion of this money did he expend ?

2. Make the whole amount which he had originally the denominator, and the amount expended the numerator, and then reduce the fraction to the lowest terms.

3. A party went on an excursion. They paid 25 cents for refreshments, 25 cents for the use of a boat, and 14 cents for two boys to row. The party consisted of a gentleman and four boys. The gentleman paid 32 cents. What proportion of the whole did he pay ?

4. The boys were of different ages. The gentleman said that he thought the oldest boy might properly pay $\frac{2}{3}$ of the rest. What part of the whole in this case would the oldest boy pay ?

5. Some boys playing near a deep brook, took off their jackets and laid them down very near the brink,

so that some money, which was in one of the pockets, slipped out into the water. There was a half dollar, a quarter of a dollar, two ten cent pieces, and three cents. The boys stuck some pitch upon the end of a long stick, and by this means got out the half dollar



and one ten cent piece. What proportion of the whole did they save in this way?

6. Afterwards one of the boys undressed himself and got into the water, and found all but one ten cent piece. What portion of the money was finally lost?

7. An old lady was knitting. Her task for the evening was to knit ten times round. There were 3 needles and 30 stitches on each. How many stitches were there in the whole task?

— oportion of her whole task was one

9. After she had knit twice round, and one needle and a half of the next, in the third round, what proportion of her whole task was done ?

10. George calculated how many stitches there were in going twice round, and then through one needle and a half, and formed the fraction from them, by making this last number the numerator, and the whole number of stitches the denominator. What was this fraction ?

11. His father wished him to see if he could reduce the fraction to any lower terms, so as to express it more briefly.

12. What proportion of the work would be done after the old lady had knit to the end of the first needle in the fourth round? Give the answer in the lowest terms.

13. What proportion after she had knit to the middle of the second needle in the eighth round ?

14. What proportion of the whole would 1 stitch be ?

15. What proportion would 5 stitches be ?

16. A boy once had to drive a cow in the country from one town to another, a distance of 60 miles. He expected it would take several days. As he went along, he amused himself by calculating what proportion of his journey he had taken from time to time, as he reached the several villages where he could ascertain his distance. At the first village he had gone 5 miles. He wanted to know what proportion this was of the whole distance. He accordingly made the whole number of miles the denominator, a

number of miles he had gone the numerator, and then reduced the fraction correctly to the lowest terms. What was his answer ?

17. What proportion of the whole distance had he gone when he reached the second village, which was 5 miles beyond the first ?

18. At the third village, which was 6 miles from the second ?

19. At the fourth village, which was 1 mile farther ?

20. When he had gone 45 miles in all, what proportion of his journey had he accomplished ?

21. When he had gone 58 miles what proportion of the distance remained ?

22. A gentleman gave his boys a piece of twine to be divided equally among them. There were 5 boys. The boys had a yardstick which was divided into inches. There were 36 inches in the yard, and of course each inch was $\frac{1}{36}$. They measured the twine, and found it 3 yards and $\frac{2}{3}$, that is, $3\frac{2}{3}$ of a yard. They reduced this mixed number to an improper fraction, and then divided it by 5 to get each one's share; what was the result ?

23. How much would have been each one's share, if there had been $4\frac{1}{3}$ of a yard of the twine ?

24. How much if there had been $1\frac{4}{8}$?

25. Their father stood by observing them as they made their calculation, and he noticed that it so happened that they could divide their fraction by dividing the numerator. Wishing to see whether they would remember how to divide the fraction, when they could divide the numerator, he asked them what would

have been each one's share, if there had been 1 yard and $\frac{7}{8}$. What should have been their answer?

26. Suppose there had been $3\frac{3}{8}$?

27. Suppose there had been 25 yards and $\frac{1}{3}$, how much would each one's share have been, expressed in thirty-sixths?

28. In this last case the fraction expressing each one's share is an improper fraction; reduce it to a whole or mixed number.

29. A merchant owned $\frac{3}{4}$ of a ship. He sold $\frac{1}{2}$ of his share to his brother; what was his brother's share, reduced to its lowest terms?

30. What part of an hour is 45 minutes, expressed in a fraction reduced to its lowest terms?

III. ADDITION OF FRACTIONS.

LESSON I.

CASE I.—WHEN THE DENOMINATORS OF THE FRACTIONS ARE THE SAME.

Explain to the pupil that when two or more fractions have the same denominator, it is very easy to add them. Thus $\frac{2}{3}$ and $\frac{1}{3}$ obviously make $\frac{3}{3}$. And so $\frac{3}{4}$ and $\frac{1}{4}$ make $\frac{4}{4}$. But where the denominators are unlike, it is much more difficult to add them. Thus, though $\frac{2}{3}$ and $\frac{1}{3}$ may be easily added, and also $\frac{2}{4}$ and $\frac{1}{4}$, it is much more difficult to add $\frac{2}{3}$ and $\frac{1}{4}$. The result will not be $\frac{3}{7}$ nor $\frac{3}{12}$, but something different from either, which it will require some calculation to ascertain.

Some of the pupils will at first be disposed to say that $\frac{2}{3}$ and $\frac{1}{3}$ make $\frac{3}{6}$, adding the denominators as well as the numerators. This, however, they may be easily led to see, is as absurd as to say that 2 half dollars and 3 half dollars added together make 5 quarter dollars. Three things and two things of any kind make five things of that same kind always, whether the denomination be wholes, or halves, or sixths, or anything else. This must be fully explained until they all see clearly that when the denominators of the fractions are alike, they only add the numerators, and preserve the denominator unchanged.

QUESTIONS.

How much are 4 apples and 3 apples added together? How much are 4 yards and 3 yards? 4 half

dollars and 3 half dollars? 4 sevenths and 3 sevenths?

Will 3 sevenths and 4 fifths make 7 sevenths? Will they make 7 fifths? Will 3 sevenths and 4 sevenths make 7 sevenths? Can you add $\frac{1}{30}$ and $\frac{2}{30}$ in their present form? What will be the sum? Can you add $\frac{2}{5}$ and $\frac{3}{5}$? Can you add $\frac{2}{10}$ and $\frac{4}{10}$? What will be the sum?

What is the sum of $\frac{2}{5}$ and $\frac{3}{5}$? Of $\frac{1}{5}$ and $\frac{7}{5}$? Of $\frac{2}{5}$ and $\frac{3}{5}$? Of $\frac{1}{100}$ and $\frac{99}{100}$? Of $\frac{1}{100}$ and $\frac{1}{100}$? Of $\frac{2}{100}$, and $\frac{1}{100}$? Of $\frac{1}{2}$, $\frac{2}{2}$, and $\frac{1}{2}$?

When you have to add fractions, is it the numerators or the denominators which must be alike to enable you to add them easily? How do you add them in such a case? What will always be the denominator of the answer?

Work upon the Slate.

$$(1.) \quad \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

The following examples are to be added in the same manner, the answer to be given without any reduction.

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{1}{2}$, and $\frac{1}{2}$.

- | | |
|---|---|
| 2. $\frac{1}{30} + \frac{7}{30}$ | 7. $\frac{2}{30} + \frac{5}{30} + \frac{8}{30}$ |
| 3. $\frac{2}{30} + \frac{7}{30} + \frac{6}{30}$ | 8. $\frac{1}{10} + \frac{3}{10} + \frac{7}{10}$ |
| 4. $\frac{2}{5} + \frac{13}{5} + \frac{1}{5}$ | 9. $\frac{1}{2} + \frac{3}{2}$ |
| 5. $\frac{1}{5} + \frac{7}{5}$ | 10. $\frac{7}{8} + \frac{1}{8}$ |
| 6. $\frac{2}{3} + \frac{3}{3}$ | 11. $\frac{1}{2} + \frac{1}{2}$ |

12. $\frac{3}{4} + \frac{3}{8}$

13. $\frac{1}{2} + \frac{1}{8}$

14. $\frac{3}{8} + \frac{1}{2} + \frac{1}{8}$

15. $\frac{1}{6} + \frac{2}{6} + \frac{1}{6}$

16. $\frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

17. $\frac{1}{6} + \frac{1}{6}$

18. $\frac{2}{3} + \frac{1}{3}$

19. $\frac{1}{2} + \frac{1}{2}$

20. $\frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

21. $\frac{1}{2} + \frac{1}{2}$

LESSON II.

When the answers to the examples given in the last lesson are reported in the class, the attention of the pupils is to be called to the fact that these answers are, in many cases, susceptible of reduction to a simpler form. For instance, the answer to the second example is $\frac{1}{2}$. This is an improper fraction, which may be reduced in the manner heretofore explained, Lesson XIII., page 71, to the form of $2\frac{1}{2}$, which is a simpler and more intelligible form. So the third example gives for the answer $\frac{1}{2}$, which, though not an improper fraction may be reduced to lower terms by dividing both numerator and denominator by the same number, viz., 5, as explained in Lesson X., page 64. This will give $\frac{2}{5}$ for the answer in its simplest form. Let the teacher give out some of the questions of the above-mentioned lessons, to be performed in the class, by way of review, in order to be sure that the processes are distinctly recalled to the mind. Then the following examples may be assigned, with directions that the answers should be reduced to their lowest terms whenever they are susceptible of such reduction. Explain also that all such fractions as $\frac{7}{7}$, and $\frac{1}{1}$, and $\frac{1}{1}$, &c., are of course, in value, equal to 1, and may be so written at once, without going through the formality of a reduction.

Work upon the Slate.

(22.) $\frac{1}{2} + \frac{3}{2} + \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2}$ Ans.

(23.) $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$ Ans.

EXAMPLES.

- | | |
|---|--|
| 22. $\frac{1}{25} + \frac{3}{25} + \frac{1}{25}$ | 33. $\frac{7}{8} + \frac{1}{8}$ |
| 23. $\frac{7}{10} + \frac{9}{10}$ | 34. $\frac{1}{25} + \frac{2}{25} + \frac{2}{25}$ |
| 24. $\frac{4}{9} + \frac{5}{9} + \frac{2}{9}$ | 35. $\frac{1}{17} + \frac{1}{17}$ |
| 25. $\frac{3}{7} + \frac{1}{7}$ | 36. $\frac{9}{10} + \frac{9}{10} + \frac{2}{10}$ |
| 26. $\frac{1}{4} + \frac{2}{4} + \frac{2}{4}$ | 37. $\frac{3}{7} + \frac{4}{7}$ |
| 27. $\frac{3}{8} + \frac{3}{8}$ | 38. $\frac{1}{15} + \frac{2}{15}$ |
| 28. $\frac{9}{100} + \frac{3}{100} + \frac{5}{100}$ | 39. $\frac{1}{100} + \frac{1}{100}$ |
| 29. $\frac{1}{20} + \frac{1}{20}$ | 40. $\frac{1}{8} + \frac{1}{8}$ |
| 30. $\frac{7}{80} + \frac{1}{80}$ | 41. $\frac{1}{4} + \frac{1}{4}$ |
| 31. $\frac{1}{110} + \frac{2}{110}$ | 42. $\frac{1}{30} + \frac{2}{30}$ |
| 32. $\frac{1}{10} + \frac{9}{10}$ | 43. $\frac{4}{50} + \frac{4}{50}$ |

LESSON III.

CASE II.—WHEN ONE OF THE DENOMINATORS IS A
MULTIPLE OF THE OTHERS.

When the two fractions have not a common denominator, that is, a denominator common to both, they may sometimes easily be made alike by *expanding* one of the fractions so as to make its denominator correspond with that of the other, in the manner taught in Lesson I., page 49. Thus, suppose we have $\frac{1}{2}$ to be added to $\frac{2}{3}$. We observe that the denominators are unlike, and we therefore cannot add the fractions in their present form; for we cannot add halves to fourths easily. But we can change the half into fourths by expanding the fraction by 2, which makes it $\frac{2}{4}$. We have now $\frac{2}{4}$ and $\frac{2}{3}$, which makes $\frac{4}{3}$. When, therefore, we have two fractions of different denominators, if we observe that by expanding one we can make its denominator like the other, we do so, and then add without difficulty.

This should be explained fully by means of illustrations on the

blackboard, and the proper mode of arranging the work upon the slate should be also fully shown—the fractions being written in the original form in one line, and in the altered form in the line below; and the pupil being specially taught that the value of the fraction altered is not at all changed by the change in its form.

QUESTIONS.

By what factor will you expand $\frac{1}{4}$ to make it correspond in denominator to $\frac{1}{3}$? How will you expand $\frac{1}{4}$ to make it correspond with $\frac{1}{5}$? How will you expand the fraction $\frac{1}{4}$ to make it correspond with $\frac{7}{8}$? The fraction $\frac{1}{4}$ to correspond with $\frac{4}{5}$? The fraction $\frac{1}{4}$ to correspond with $\frac{5}{8}$? $\frac{1}{4}$ to correspond with $\frac{3}{8}$? $\frac{1}{4}$ to correspond with $\frac{2}{5}$?

After going round the class a few times with questions like the above, the pupils may all be led to observe that in order to make the fractions correct in all such cases, they have only to consider what number the denominator of one fraction must be multiplied by to make it equal to that of the other, and then to preserve the value of the fraction unaltered, the numerator must be multiplied by the same. Explain to them also, that when the larger denominator is such that it can be exactly produced by multiplying the other, it is called a *multiple* of it. Thus, in the two fractions $\frac{2}{3}$ and $\frac{1}{15}$, the second denominator is a multiple of the first, as 15 can be produced by multiplying 5 by 3. Whenever, therefore, the denominator of one fraction is a multiple of the denominator of the other, the latter can be expanded to correspond with the former as above shown. This, they must see, is not *multiplying* the fraction, but only *expanding* its form, leaving the value the same as before. In the following examples the answers are to be reduced to their simplest terms :—

Work upon the Slate.

$$\begin{aligned} (44.) \quad & \frac{4}{3} + \frac{3}{10} \\ & \frac{8}{15} + \frac{3}{10} = 1\frac{1}{10} \text{ Ans.} \\ (45.) \quad & \frac{7}{8} + \frac{1}{8} \\ & 1\frac{1}{8} + \frac{1}{8} = 1\frac{2}{8} \text{ Ans.} \end{aligned}$$

EXAMPLES.

44. $\frac{4}{3} + \frac{3}{10}$	55. $\frac{5}{7} + \frac{5}{8}$
45. $\frac{7}{8} + \frac{1}{8}$	56. $1\frac{5}{8} + \frac{1}{8}$
46. $\frac{5}{6} + \frac{1}{8}$	57. $\frac{3}{5} + \frac{1}{6}$
47. $\frac{3}{5} + \frac{5}{8}$	58. $1\frac{1}{6} + \frac{3}{8}$
48. $\frac{1}{3} + \frac{1}{8}$	59. $\frac{4}{6} + \frac{2}{3}$
49. $1\frac{4}{8} + \frac{5}{8}$	60. $\frac{5}{8} + \frac{2}{4}$
50. $\frac{1}{7} + \frac{2}{4}$	61. $\frac{4}{9} + \frac{1}{3}$
51. $\frac{3}{5} + \frac{1}{3}$	62. $1\frac{3}{7} + \frac{1}{3}$
52. $\frac{1}{2} + \frac{1}{30}$	63. $\frac{1}{2} + \frac{1}{8}$
53. $1\frac{2}{3} + \frac{1}{8}$	64. $\frac{1}{2} + \frac{7}{8}$
54. $\frac{1}{2} + \frac{2}{3}$	65. $\frac{2}{3} + \frac{1}{2}$

LESSON IV.

Sometimes there will be several fractions of different denominators, and yet they can all be made to correspond by expanding some by one factor and some by another. To illustrate this, let the teacher write upon the blackboard the expression

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{5} + 1\frac{1}{8} + \frac{1}{4}$$

Then ask which is the greatest of the denominators, and whether the others can be expanded so as to correspond with it, and how.

factor the first must be expanded, and the second, and the third, &c. Then write the expression again, giving to the fractions their new forms, thus :—

$$\frac{9}{18} + \frac{6}{18} + \frac{12}{18} + \frac{1}{18} + \frac{2}{18}$$

The fractions being now all of one denomination, viz., eighteenths, may easily be added by adding the numerators.

If the above example seems too long to be clearly understood, repeat the explanation with simpler numbers, as

$$\frac{1}{2} + \frac{1}{2} + \frac{2}{2}$$

which will give

$$\frac{2}{2} + \frac{2}{2} + \frac{2}{2} = \frac{6}{2} = 3$$

EXAMPLES.

- | | |
|--|---|
| 66. $\frac{1}{2} + \frac{1}{6} + \frac{1}{3}$ | 76. $\frac{5}{6} + \frac{2}{3} + \frac{1}{2}$ |
| 67. $\frac{3}{4} + \frac{1}{4} + \frac{1}{2}$ | 77. $\frac{2}{3} + \frac{1}{6} + \frac{1}{6}$ |
| 68. $\frac{3}{8} + \frac{1}{4} + \frac{1}{2}$ | 78. $\frac{1}{6} + \frac{2}{3} + \frac{1}{2}$ |
| 69. $\frac{2}{3} + \frac{1}{6} + \frac{2}{3}$ | 79. $\frac{1}{6} + \frac{2}{3} + \frac{1}{2}$ |
| 70. $\frac{2}{7} + \frac{1}{4} + \frac{1}{7}$ | 80. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ |
| 71. $\frac{5}{6} + \frac{1}{3} + \frac{2}{3}$ | 81. $\frac{2}{3} + \frac{1}{3} + \frac{1}{3}$ |
| 72. $\frac{7}{8} + \frac{1}{2} + \frac{1}{8}$ | 82. $\frac{2}{3} + \frac{1}{6} + \frac{1}{6}$ |
| 73. $\frac{9}{10} + \frac{1}{10} + \frac{1}{10}$ | 83. $\frac{1}{2} + \frac{1}{6} + \frac{1}{6}$ |
| 74. $\frac{1}{2} + \frac{1}{2} + \frac{2}{2}$ | 84. $\frac{1}{2} + \frac{1}{6} + \frac{1}{6}$ |
| 75. $\frac{2}{3} + \frac{1}{3} + \frac{1}{3}$ | 85. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ |

LESSON V.

CASE III.—WHEN NEITHER OF THE DENOMINATORS IS A MULTIPLE OF THE OTHERS.

be shown that he cannot always add fractions as
 ing examples ; for since it will not always hap-

pen that one of the denominators is a multiple of the others, he cannot always make the fractions correspond in their denominations by expanding one of them. This can only be done in cases where the largest of the two denominators can be produced by simply multiplying the smallest, which is the case in the examples given above. But sometimes we have fractions like the following: $\frac{2}{5}$ and $\frac{3}{7}$. Now these fractions cannot be made to correspond by expanding either one, for 7 is not a multiple of 5. But they may be made to do so by expanding both. Thus, if we expand $\frac{2}{5}$ by 7, it becomes $\frac{14}{35}$, and expanding $\frac{3}{7}$ by 5, it becomes $\frac{15}{35}$. These may now easily be added. The work should be arranged upon the blackboard by the teacher, thus:—

$$\begin{array}{r} \frac{2}{5} + \frac{3}{7} \\ \frac{14}{35} + \frac{15}{35} = \frac{29}{35} \end{array}$$

It will be well not to explain immediately to the class how you determine by what numbers to expand these fractions in order to make them correspond; but give them several examples upon the blackboard, as $\frac{1}{3}$ and $\frac{2}{3}$, which you expand to fifteenths; $\frac{1}{2}$ and $\frac{2}{3}$, &c., $\frac{2}{3}$ and $\frac{1}{4}$, &c. The teacher can write an example of this kind upon the blackboard, and let the class see if they can tell how to expand both fractions so as to make them correspond. By thus exciting their curiosity, the announcement at last of the principle, viz., *that you expand each fraction by the denominator of the other*, will interest them more strongly, and make a more lasting impression. Of course, if any of the class have become so much interested in this study as to read these directions themselves in anticipation, they will have discovered the principle before coming to the class. Then assign the following examples to be added—the fractions to be prepared by expanding each by the denominator of the other, which will always make them correspond. The answers are to be reduced to their lowest terms.

EXAMPLES.

86. $\frac{1}{4} + \frac{1}{4}$

87. $\frac{1}{4} + \frac{3}{4}$

88.	$\frac{1}{2} + \frac{2}{3}$	97.	$\frac{4}{5} + \frac{1}{2}$
89.	$\frac{1}{2} + \frac{1}{3}$	98.	$\frac{7}{8} + \frac{1}{2}$
90.	$\frac{1}{2} + \frac{2}{3}$	99.	$\frac{2}{3} + \frac{2}{3}$
91.	$\frac{1}{2} + \frac{1}{3}$	100.	$\frac{2}{3} + \frac{1}{2}$
92.	$\frac{2}{3} + \frac{1}{3}$	101.	$\frac{1}{2} + \frac{1}{2}$
93.	$\frac{2}{3} + \frac{2}{3}$	102.	$\frac{7}{8} + \frac{1}{2}$
94.	$\frac{7}{8} + \frac{7}{8}$	103.	$\frac{4}{5} + \frac{2}{3}$
95.	$\frac{2}{3} + \frac{4}{5}$	104.	$\frac{1}{2} + \frac{2}{3}$
96.	$\frac{1}{2} + \frac{1}{3}$	105.	$\frac{4}{5} + \frac{1}{2}$

LESSON VI.

When there are more than two fractions, and neither denominator is a multiple of the others, they may always be made to correspond by expanding each one by all the denominators except its own. This will not alter the value of any fraction, but it will only change its form, and in the new forms the denominators will always be alike, and then the fractions can easily be added.

It will be necessary to explain this process very slowly and distinctly upon the blackboard, by writing three fractions, as $\frac{2}{3}$, $\frac{1}{2}$, $\frac{2}{5}$, and then showing the class that they can be reduced to a common denominator by expanding $\frac{2}{3}$ by 3 and 7, and $\frac{1}{2}$ by 5 and 7, and $\frac{2}{5}$ by 5 and 3. The work would be arranged thus:—

$$\frac{2}{3} + \frac{1}{2} + \frac{2}{5} = \frac{2 \cdot 7}{105} + \frac{1 \cdot 7}{105} + \frac{2 \cdot 3}{105} = \frac{14}{105} = 1\frac{4}{105}$$

It will be well to show the class that this process is the one described in many books on Arithmetic by the following rule, viz.: Multiply each numerator into all the denominators except its own for a new numerator, and all the denominators together for a common denominator. It is perhaps better, however, to have the operation presented more prominently to the mind, in the light in which it stands, if each fraction is considered as *expanded* by all the other except its own, as thus they see more easily that the

new fractions obtained are not new *values*, but only new forms. It is well, also, that they should arrange the fractions in two lines as above, placing the new form, in each case, under the old one, in order that they keep in mind distinctly, that in the second line each fraction only takes the place of its representative and equal in the line above.

They may be taught also, that in expanding each fraction, it is immaterial whether they multiply each term by the two denominators successively, or whether they first combine the two denominators by multiplying them together, and then afterwards expanding the fraction by the result. Thus, in the case above given, they may first expand $\frac{1}{2}$ by 3, which gives $\frac{3}{6}$, and then by 7, which gives $\frac{21}{42}$, or they may first multiply 3 and 7 together, and then expand $\frac{1}{2}$ by the result, viz., 21, which will give $\frac{21}{42}$ as before. This process of altering the forms of fractions so as to make the denominators alike, without altering their value, is called *reducing them to a common denominator*.

Assign the following fractions to be reduced to a common denominator and added; the result to be reduced to the simplest terms.

Work upon the Slate.

$$(106.) \quad \frac{3}{4} + \frac{1}{2} + \frac{2}{3} \\ \frac{9}{12} + \frac{6}{12} + \frac{8}{12} = \frac{23}{12} = 1\frac{11}{12} \text{ Ans.}$$

EXAMPLES.

- | | | | |
|--|---|------|---|
| 106. $\frac{3}{4} + \frac{1}{2} + \frac{2}{3}$ | | | |
| 107. | $\frac{2}{3} + \frac{1}{4} + \frac{1}{2}$ | 115. | $\frac{4}{5} + \frac{7}{10}$ |
| 108. | $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ | 116. | $\frac{3}{8} + \frac{1}{4}$ |
| 109. | $\frac{7}{10} + \frac{1}{2} + \frac{1}{5}$ | 117. | $\frac{2}{3} + \frac{1}{2} + \frac{1}{4}$ |
| 110. | $\frac{9}{10} + \frac{7}{8}$ | 118. | $\frac{1}{2} + \frac{2}{3} + \frac{1}{4}$ |
| 111. | $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$ | 119. | $\frac{3}{10} + \frac{4}{15} + \frac{1}{2}$ |
| 112. | $\frac{1}{7} + \frac{3}{8}$ | 120. | $\frac{1}{8} + \frac{1}{7} + \frac{1}{5}$ |
| 113. | $\frac{2}{3} + \frac{3}{7} + \frac{1}{2}$ | 121. | $\frac{3}{4} + \frac{1}{8} + \frac{2}{10}$ |
| 114. | $\frac{7}{10} + \frac{7}{15} + \frac{1}{3}$ | 122. | $\frac{5}{7} + \frac{1}{6} + \frac{1}{3}$ |

$$\begin{array}{ll}
 123. & \frac{1}{4} + \frac{3}{8} + \frac{5}{8} \\
 124. & \frac{5}{8} + \frac{1}{4} + \frac{1}{8}
 \end{array}
 \quad
 \begin{array}{ll}
 125. & \frac{3}{8} + \frac{3}{8} + \frac{1}{4} \\
 126. & \frac{1}{4} + \frac{5}{8} + \frac{1}{8}
 \end{array}$$

LESSON VII.

Explain that in the preceding cases, the new denominator is always a multiple of all those which were originally given. Take, for instance, the last example.

$$\frac{1}{4} + \frac{5}{8} + \frac{1}{8} = \frac{3}{216} + \frac{9}{216} + \frac{3}{216} = \frac{15}{216} = \frac{5}{72} = \frac{5}{72} \text{ Ans.}$$

In this case, 216, the new denominator, is a multiple of 4, of 9, and of 6, the denominators originally given. Such a number, which is a multiple of several other numbers in common, is called a *common multiple*. In adding fractions whose denominators are unlike, we make them alike by expanding each fraction, and the new denominator so obtained will be a common multiple of all the others.

But by the method above given, the common multiple is larger than is necessary. The number 216 is indeed a common multiple of 4, 9, and 6, but there is a smaller number which is a common multiple, viz., 36. For 36 is 9 times the first denominator, 4 times the second, and 6 times the third. Of course, if we expand the first fraction by 9, the second by 4, and the third by 6, the denominators will be alike, thus:—

$$\frac{1}{4} + \frac{5}{8} + \frac{1}{8} = \frac{9}{36} + \frac{5}{36} + \frac{6}{36} = \frac{20}{36} = \frac{5}{9} \text{ Ans.}$$

The answer is thus the same as before, though we obtain it by a shorter process, from knowing a smaller common multiple than that obtained by multiplying all the denominators together.

Explain to the pupils, too, that when they know what the least common multiple is, as that is to be the new denominator of the sum, they may write it at once as many times as they have

fractions to be reduced, with a short line above it, and then afterwards calculate the numerators and insert them; thus :—

$$\frac{1}{4} + \frac{4}{9} + \frac{1}{3}$$

And in order to calculate the numerators in each case, they have only to consider first what each denominator had to be multiplied by to give 36, and then multiply each numerator by the same. Thus, the first numerator must be multiplied by 9, because 36 is 9 times 4. The second must be multiplied by 4, because 36 is 4 times 9, &c. This is obviously true, since both numerator and denominator must be multiplied by the same number, in order to expand the fraction without altering the value.

Say also to the class, that in the next lesson they will be taught how to *obtain* the smallest common multiple, but at present they are only to learn how to expand the fractions when they know what the smallest common multiple is, which is to make the new denominator.

In the following examples then, they are to expand the fractions to such forms as shall make the denominators correspond with the least common multiples given. The common multiple is called sometimes *the common denominator*, for it becomes the new denominator of all the fractions. The work is to be arranged thus :—

127. Add $\frac{2}{3}$, $\frac{2}{3}$, and $\frac{2}{15}$; com. mult. of the denominator 30.

Work upon the Slate.

$$(127.) \quad \frac{2}{3} + \frac{2}{3} + \frac{2}{15} \\ \frac{14}{15} + \frac{14}{15} + \frac{2}{15} = \frac{30}{15} = 2 \text{ Ans.}$$

EXAMPLES.

128. Add $\frac{1}{3}$, $\frac{1}{3}$ and $\frac{1}{3}$ com. den. 18.

129.	Add	$\frac{4}{5}$	$\frac{1}{15}$	and	$\frac{1}{15}$	com. den.	70.
130.	"	$\frac{1}{3}$	$\frac{1}{4}$	and	$\frac{1}{15}$	"	60.
131.	"	$\frac{2}{3}$	$\frac{2}{3}$	and	$\frac{1}{15}$	"	45.
132.	"	$\frac{1}{3}$	$\frac{1}{6}$	and	$\frac{1}{15}$	"	60.
133.	"	$\frac{4}{5}$	$\frac{1}{3}$	and	$\frac{1}{15}$	"	30.
134.	"	$\frac{1}{3}$	$\frac{1}{15}$	and	$\frac{1}{15}$	"	60.
135.	"	$\frac{4}{5}$	$\frac{1}{15}$	and	$\frac{1}{15}$	"	140.
136.	"	$\frac{1}{3}$	$\frac{2}{15}$	and	$\frac{1}{15}$	"	144.
137.	"	$\frac{1}{3}$	$\frac{1}{3}$	and	$\frac{1}{3}$	"	24.
138.	"	$\frac{1}{4}$	$\frac{2}{7}$	and	$\frac{2}{28}$	"	140.
139.	"	$\frac{1}{4}$	$\frac{1}{3}$	and	$\frac{7}{12}$	"	60.
140.	"	$\frac{1}{2}$	$\frac{1}{3}$	and	$\frac{2}{6}$	"	6.
141.	"	$\frac{1}{7}$	$\frac{1}{15}$	and	$\frac{1}{105}$	"	126.
142.	"	$\frac{2}{3}$	$\frac{2}{7}$	and	$\frac{2}{21}$	"	28.
143.	"	$\frac{1}{15}$	$\frac{1}{15}$	and	$\frac{1}{15}$	"	720.
144.	"	$\frac{4}{5}$	$\frac{2}{3}$	and	$\frac{1}{3}$	"	30.
145.	"	$\frac{5}{6}$	$\frac{1}{15}$	and	$\frac{1}{15}$	"	90.
146.	"	$\frac{2}{7}$	$\frac{2}{3}$	and	$\frac{1}{42}$	"	840.
147.	"	$\frac{1}{15}$	$\frac{2}{3}$	and	$\frac{1}{15}$	"	180.

•

LESSON VIII.

In the preceding lesson the smallest number which will make a common denominator is given to the pupils. Say to them that they are now to learn how to discover it for themselves; and then explain to them the usual process of finding it, by dividing the given denominators in succession by any numbers which will exactly divide two or more of them, and then multiplying together the divisors and the incommensurable numbers which remain. Suppose it to be required, for example, to find the least common multiple of the denominators of the following fractions, $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{1}{15}$.

First write the denominators in a line and divide as above directed.

$$\begin{array}{r} 2) 6 \quad 9 \quad 10 \\ \hline 3) 3 \quad 9 \quad 5 \\ \hline 1 \quad 3 \quad 5 \end{array}$$

The divisors are 2 and 3, and the incommensurable numbers remaining are 3 and 5. These, multiplied together, give 90, the smallest common multiple—which was required.

It will be well to allow the pupils to perform several of the following examples together, in the class, while the teacher is present to aid them—continuing the exercise until all are well acquainted with the process. Then let the work which they have performed be effaced, and let the class return to their seats, and attempt to perform the examples alone. Before going there, however, their attention should be called again distinctly to the several steps of the process.

1. Write the denominators in a line.
2. If any two or more of them are divisible by any number, as 2, 3, 4, &c., divide them and bring down the quotients, and also the numbers not divided, to the line below.
3. Repeat this operation as long as any two or more are divisible.
4. Multiply all the divisors, and the numbers remaining in the lower line together, and the product will be the smallest common multiple.

The class are not to commit the words of the above directions to memory, but endeavor, on the other hand, to impress strongly upon their minds the several steps described by them, as *things to be done*. They are also to be reminded that the only occasion which we have, in general, to find the least common multiple of two or more numbers, is in order to find *the smallest denominator to which any given fractions can be expanded*, as a preparation for adding them.

148. Find the least common multiple of the denominators of the following fractions ;— $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{12}$.

Work upon the Slate.

$$\begin{array}{r}
 (148.) \quad 2) 5 \quad 4 \quad 10 \\
 \hline
 5) 5 \quad 2 \quad 5 \\
 \hline
 1 \quad 2 \quad 1 \\
 2 \times 5 \times 2 = 20 \text{ Ans.}
 \end{array}$$

EXAMPLES.

Find the least common multiple of the denominators of the following fractions.

- | | | | | | | | |
|------|---------------|----------------|------------------|------|----------------|----------------|------------------|
| 149. | $\frac{2}{3}$ | $\frac{5}{6}$ | $\frac{1}{12}$. | 154. | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$. |
| 150. | $\frac{2}{3}$ | $\frac{1}{12}$ | $\frac{1}{18}$. | 155. | $\frac{1}{2}$ | $\frac{1}{12}$ | $\frac{1}{18}$. |
| 151. | $\frac{2}{3}$ | $\frac{1}{4}$ | $\frac{1}{12}$. | 156. | $\frac{1}{12}$ | $\frac{1}{18}$ | $\frac{1}{4}$. |
| 152. | $\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{1}{4}$. | 157. | $\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{1}{4}$. |
| 153. | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{12}$. | 158. | $\frac{1}{2}$ | $\frac{1}{12}$ | $\frac{1}{18}$. |

LESSON IX.

The pupils must now be reminded that as they learned in Lesson VII. how to add fractions when the least common multiple of the denominators was given, and in Lesson VIII. how to find the least common multiple, they are now prepared to combine the two operations,

we, when there are no two denominators which can be at multiply them together as they are.

by finding the multiple, and then adding the fractions. In the following examples they may accordingly do this, and then reduce the fractions to the lowest terms. The work will appear thus: the example taken being the first of those given below.

Work upon the Slate.

(159.) 3) 9 3 6 denominators.

$$\begin{array}{r} 3 \quad 1 \quad 2 \\ \hline \end{array}$$

$$3 \times 3 \times 2 = 18 \text{ com. mult.}$$

$\frac{1}{3} + \frac{2}{3} + \frac{2}{3}$ fractions to be added.

$$\frac{1}{18} + \frac{12}{18} + \frac{12}{18} = \frac{25}{18} = 1\frac{7}{18} \text{ Ans.}$$

EXAMPLES.

159. Add $\frac{1}{3}$, $\frac{2}{3}$ and $\frac{2}{3}$. 164. Add $\frac{1}{18}$, $\frac{2}{18}$ and $\frac{1}{3}$.
 160. " $\frac{2}{3}$, $\frac{1}{3}$ and $\frac{1}{18}$. 165. " $\frac{2}{3}$, $\frac{1}{3}$ and $\frac{1}{18}$.
 161. " $\frac{1}{4}$, $\frac{2}{7}$ and $\frac{1}{28}$. 166. " $\frac{1}{18}$, $\frac{1}{18}$ and $\frac{1}{3}$.
 162. " $\frac{7}{18}$, $\frac{2}{3}$ and $\frac{4}{18}$. 167. " $\frac{2}{3}$, $\frac{1}{36}$ and $\frac{1}{36}$.
 163. " $\frac{1}{4}$ and $\frac{1}{18}$. 168. " $\frac{2}{18}$ and $\frac{1}{3}$.

When the work of the above examples is presented for examination, the teacher should particularly observe whether it is arranged in a neat and symmetrical manner upon the slate, corresponding to the model given above.

LESSON X.

The teacher should now explain fully to the class the distinction between a common *multiple* and a common *measure*. A common measure of any two numbers, as explained on page 59, is a number which will *divide them both*; while a common multiple is one which *they will both divide*. Thus 3 is a common measure of 9 and 12,

180.	Add	$\frac{4}{5}$,	$\frac{1}{16}$,	$2\frac{1}{16}$	and	$\frac{3}{8}$.
181.	"	$\frac{1\frac{1}{2}}{16}$,	$\frac{1}{16}$,	$3\frac{3}{8}$	and	$\frac{1}{2}$.
182.	"	$\frac{1}{16}$,	$\frac{1}{16}$,	$\frac{3}{8}$ $\frac{5}{8}$	and	2.
183.	"	$\frac{1}{14}$,	$\frac{1}{7}$,	$\frac{2}{14}$	and	$\frac{1}{5}$.
184.	"	$\frac{1}{9}$,	$\frac{1}{16}$,	$\frac{1}{36}$	and	$\frac{2}{3}$.
185.	"	$\frac{2}{5}$,	$\frac{1}{16}$,	$\frac{1}{5}$	and	$\frac{1}{16}$.
186.	"	$\frac{1}{4}$,	$\frac{1}{2}$		and	$\frac{3}{4}$.
187.	"	$\frac{4}{8}$,	$\frac{1}{16}$		and	$\frac{1}{5}$.

MISCELLANEOUS EXAMPLES.

1. A merchant owned $\frac{3}{16}$ of a ship. His brother owned $\frac{1}{16}$ and he bought one-half of his brother's share. How much did he then own in all?

2. A boy wanted to make a fishing line. He had four pieces of twine. One was $2\frac{1}{2}$ yards long, another $1\frac{1}{4}$ yards, the third $\frac{7}{8}$ of a yard, and the fourth $2\frac{3}{16}$ of a yard. How long a line would they all make?

3. Some children went on a party to gather berries. There were four in the party, and they agreed to divide their berries equally on their return. Mary, one of the party, gave one-third of her share to her cousin on her return. What proportion of the whole did her cousin receive, supposing she was one of the party?

4. What proportion of the whole would she have received if she had not been one of the party?

5. One-third of the soldiers of an army died upon a march, and one-fifth of the remainder were killed in battle. What proportion of the whole army died?

6. A man taking a journey on foot, traveled $\frac{1}{4}$ part

of the way and then found that he had lost his pocket-book. He went back $\frac{2}{3}$ of the way he had traveled to find it. What portion of the whole distance which he had traveled did he go over twice?

7. Which is greatest, $\frac{2}{3}$ of $\frac{1}{3}$ or $\frac{2}{3}$ of $\frac{1}{4}$?

In such cases as this reduce the fractions to a common denominator, and then they can easily be compared.

8. Which is greatest, $\frac{1}{3}$ of $\frac{1}{16}$ of a ship, or $\frac{2}{3}$ of $\frac{1}{32}$ of it?

9. A man of fortune died, leaving his property in equal shares to his 5 sons. The second one died, and his share was divided among his children, 3 in number. What proportion of the whole fortune did each of these three children receive?

10. A gentleman had a house which he wished to carpet. There were four rooms which were to be carpeted alike. One required $31\frac{3}{4}$ yards. The second, $14\frac{1}{2}$ yards, the third, $45\frac{1}{2}$, and the fourth, $28\frac{3}{4}$ yards. What was the whole amount required for these four rooms?

11. John had some marbles in a bag. He gave half of them to James. James gave $\frac{2}{3}$ of those which he received to Moses, and Moses lost $\frac{1}{3}$ of them. What portion of the original number were lost?

12. What proportion of the cost of a house is in the walls and roof, if the walls cost $\frac{1}{3}$ of the whole and the roof $\frac{2}{3}$?

13. Two men owned a field in common. A owned $\frac{2}{3}$ and B $\frac{1}{3}$. A sold $\frac{2}{3}$ of his to C. Which then owned the greatest share, C or B?

14. Two boys undertook to fill up a quagmire in



their father's field for a dollar. They had two wheelbarrows, one of which was a little larger than the other. Their father calculated that it would take 50 loads of one wheelbarrow, or 60 of the other to fill up the hole. They put in ten loads each the first afternoon, and then undertook to calculate what proportion of the work was done. What should have been the answer?

15. Had they earned more or less than one-third of the money?

IV. SUBTRACTION OF FRACTIONS.

LESSON I.

CASE I.—WHEN THE GIVEN FRACTIONS HAVE A COMMON DENOMINATOR.

EXPLAIN to the pupils that fractions may be easily subtracted as well as added, when the denominators are alike, as in that case it is only necessary to subtract one numerator from the other. Thus $\frac{1}{2}$ taken from $\frac{3}{2}$ evidently leaves $\frac{2}{2}$. And the difference between $\frac{3}{5}$ and $\frac{1}{5}$ is plainly $\frac{2}{5}$.

QUESTIONS.

How much is left if you subtract $\frac{2}{10}$ from $\frac{5}{10}$? $\frac{7}{15}$ from $\frac{9}{15}$? What is the difference between $\frac{1}{30}$ and $\frac{1}{30}$? Between $\frac{7}{10}$ and $\frac{2}{10}$? Between $\frac{9}{10}$ and $\frac{1}{10}$?

After asking questions of this kind until it is clear that all understand the general principle, that when the denominators are alike, the difference of the numerators will be the difference in the value of the fractions, explain to the class that where the numerators are large it may be necessary to write them down, one under the other, in a separate part of the slate, in order that they may perform the subtraction more conveniently. But that in such cases, when this is done, the difference of the numerators must be written over the common denominator for the difference between the fractions. Then assign the following examples :—

EXAMPLES.

- | | |
|---|--|
| 1. $\frac{1^6 7}{88} - \frac{1^2 1}{99}$. | 6. $\frac{444}{33} - \frac{313}{22}$. |
| 2. $\frac{44}{11} - \frac{35}{11}$. | 7. $\frac{33}{22} - \frac{13}{11}$. |
| 3. $\frac{133}{111} - \frac{137}{111}$. | 8. $\frac{5^6}{311} - \frac{3^6}{311}$. |
| 4. $\frac{433}{111} - \frac{133}{111}$. | 9. $\frac{77}{11} - \frac{33}{11}$. |
| 5. $\frac{2^6 9}{1111} - \frac{1^2 9}{1111}$. | 10. $\frac{14}{11} - \frac{9}{11}$. |
| 11. Subtract $\frac{1}{11}$ from $\frac{3}{11}$ of $\frac{1}{11}$. | |
| 12. From $\frac{7}{11}$ take $\frac{1}{11}$ of $\frac{3}{11}$. | |
| 13. " $\frac{17}{11}$ " $\frac{4}{11}$ of $\frac{3}{11}$. | |
| 14. " $\frac{13}{11}$ " $\frac{9}{11}$ of $\frac{1}{11}$. | |
| 15. " $\frac{343}{111}$ " $\frac{1}{11}$ of $\frac{1}{11}$. | |
| 16. Subtract $\frac{3}{11}$ of $\frac{7}{11}$ from $\frac{3}{11}$. | |
| 17. From $\frac{9}{11}$ of $\frac{1}{11}$ take $\frac{7}{11}$. | |
| 18. " $\frac{99}{111}$ take $\frac{13}{11}$ of $\frac{9}{11}$. | |
| 19. " $\frac{13}{11}$ " $\frac{3}{11}$. | |
| 20. " $\frac{33}{11}$ " $\frac{1}{11}$. | |

LESSON II.

CASE II.—WHEN THE GIVEN FRACTIONS HAVE NOT A COMMON DENOMINATOR.

The pupils will readily see that when fractions have different denominators they must be prepared as in addition, by being reduced to the same denominator, by expanding one or both of them. Then they can easily be subtracted as above. If one denominator is a multiple of the other, the one which has the smallest denominator can at once be expanded to correspond with the other, thus :—

$$\frac{4}{11} - \frac{1}{11} = \frac{3}{11} \text{ Ans.}$$

In case neither denominator is a multiple of the other, then both must be expanded to some denominator which is a multiple of both. This can always be done by expanding each fraction by the denominator of the other, thus :—

$$\frac{2}{3} - \frac{1}{4} = \frac{2 \times 4}{3 \times 4} - \frac{1 \times 3}{4 \times 3} = \frac{8}{12} - \frac{3}{12} = \frac{5}{12} \text{ Ans.}$$

Explain to the class that this process of expanding each fraction by the denominator of the other will always give a correct answer, though sometimes the fraction will be expanded more than is necessary. If they wish to expand as little as possible, they must find the smallest common multiple of the two denominators as they did in addition. Thus, in the case above given,

$$\begin{array}{r} \frac{2}{3} - \frac{1}{4} \\ 2) 6 \quad 4 \\ \hline 3 \quad 2 \end{array}$$

$$2 \times 3 \times 2 = 12 \text{ smallest common multiple.}$$

Then expanding the fraction to this denominator, we have

$$\frac{2}{3} - \frac{1}{4} = \frac{5}{12} \text{ Ans. the same as before.}$$

The class will see that this last method does not abridge the process at all, if all the work is put down upon the slate; but they may be told, that after they have had practice enough to become familiar with such operations they will perform the calculation for getting the smallest common multiple mentally, and then the work to be written would be only this :—

$$\frac{2}{3} - \frac{1}{4} = \frac{5}{12} \text{ Ans.}$$

All this must be explained very minutely and fully with other

illustrations upon the blackboard ; and while presenting such illustrations, the teacher should question the different members of the class continually upon the different steps, in order to sustain their continued attention, and to be certain that all understand fully the principles involved. In the examples which follow, the pupils may be allowed to find the smallest common multiple or expand each fraction at once by the denominator of the other, as they individually prefer. Of course, the answers will be the same in either case, provided the answer first obtained is reduced to the lowest terms.

EXAMPLES.

- | | |
|--|---------------------------------------|
| 21. Subtract $\frac{1}{2}$ from $1\frac{1}{2}$. | |
| 22. From $1\frac{2}{3}$ take $\frac{1}{2}$. | |
| 23. " $\frac{4}{5}$ " $1\frac{2}{5}$. | |
| 24. " $\frac{1}{5}$ " $1\frac{1}{5}$. | |
| 25. $\frac{1}{2}$ — $\frac{1}{3}$. | 33. $\frac{2}{3}$ — $\frac{1}{4}$. |
| 26. $\frac{2}{3}$ — $1\frac{1}{6}$. | 34. $1\frac{2}{7}$ — $1\frac{1}{6}$. |
| 27. $\frac{1}{4}$ — $1\frac{3}{4}$. | 35. $\frac{4}{5}$ — $\frac{2}{5}$. |
| 28. $1\frac{5}{6}$ — $1\frac{2}{3}$. | 36. $1\frac{7}{8}$ — $\frac{1}{2}$. |
| 29. $\frac{2}{3}$ — $\frac{1}{3}$. | 37. $1\frac{2}{3}$ — $\frac{2}{3}$. |
| 30. $\frac{2}{3}$ — $1\frac{1}{3}$. | 38. $\frac{4}{5}$ — $\frac{1}{4}$. |
| 31. $\frac{1}{2}$ — $\frac{1}{4}$. | 39. $\frac{1}{2}$ — $\frac{1}{7}$. |
| 32. $\frac{1}{2}$ — $1\frac{1}{3}$. | 40. $\frac{1}{5}$ — $1\frac{1}{5}$. |

LESSON III.

Sometimes fractions require some preparation by reducing compound ones to simple, before they can be expanded. This makes the process a little longer, but not more difficult. If there are mixed numbers also, they may be reduced to improper fractions. Mixed numbers may, however, be disposed of in another way, as ~~will~~ be shown. Illustrate and explain this fully upon the

blackboard by the following examples. Each of the forms in the second and third lines must be placed under its corresponding expression above, so as to keep the nature and design of the changes made distinctly in the mind of the pupil. Suppose it to be required to subtract $\frac{1}{2}$ of $\frac{1}{3}$ from $8\frac{1}{3}$:—

We write $8\frac{1}{3} - \frac{1}{2}$ of $\frac{1}{3}$

Reducing, we have $\frac{21}{3} - \frac{1}{3}$

Then we find by calculation that the least common multiple of 10 and 12 is 60, and expanding the fractions to this denominator and subtracting, we have

$$\frac{110}{60} - \frac{10}{60} = \frac{100}{60} = 8\frac{1}{3} \text{ Ans.}$$

When this is well understood assign the following examples to be performed in the same manner

EXAMPLES.

41. From $\frac{1}{3}$ of $\frac{2}{3}$ subtract $\frac{1}{2}$ of $\frac{1}{3}$.
42. " $3\frac{1}{2}$ " $\frac{1}{2}$ of $\frac{1}{3}$.
43. " $1\frac{1}{2}$ " $\frac{2}{3}$.
44. " $\frac{2}{3}$ of $\frac{1}{3}$ " $\frac{1}{2}$ of $\frac{2}{3}$.
45. " $7\frac{1}{2}$ " $\frac{1}{2}$ of $14\frac{1}{2}$.
46. " $3\frac{1}{2}$ " $\frac{1}{2}$ of $1\frac{1}{2}$.
47. Subtract $1\frac{1}{2}$ from $2\frac{1}{2}$.
48. " $\frac{1}{2}$ of $\frac{1}{3}$ " $\frac{1}{2}$ of $1\frac{1}{2}$.
49. " $\frac{2}{3}$ of $1\frac{1}{2}$ " $\frac{1}{2}$ of $1\frac{1}{2}$.
50. What is the value of $3\frac{1}{2} - \frac{1}{2}$?
51. " " $4\frac{7}{8} - \frac{3}{8}$?
52. " " $7\frac{1}{2} - 3\frac{1}{2}$?
53. " " $8\frac{1}{8} - \frac{3}{8}$ of $\frac{1}{2}$?
54. " " $\frac{1}{2}$ of $\frac{1}{2} - \frac{1}{2}$ of $\frac{1}{2}$?

55. Subtract $8\frac{1}{2}$ from $16\frac{3}{4}$.
 56. " $1\frac{1}{2}$ " $7\frac{3}{4}$.
 57. " $\frac{1}{2}$ of $\frac{1}{2}$ " $\frac{2}{3}$ of $\frac{3}{4}$.
 58. " $\frac{2}{3}$ " $\frac{7}{8}$.
 59. " $\frac{1}{2}$ " $\frac{1}{3}$.
 60. " $\frac{2}{3}$ of $\frac{1}{2}$ $\frac{3}{4}$ of $\frac{1}{2}$.

LESSON IV.

The pupils having thoroughly learned how to subtract in the case of mixed numbers by reducing them to improper fractions, they may now be taught that this is not always necessary. As, for example—

$$4\frac{3}{4} - \frac{1}{2}$$

In this case we have a mixed number for a minuend, and the fraction pertaining to it, viz., $\frac{3}{4}$ is greater than the $\frac{1}{2}$, which is to be subtracted. Of course, we have only to subtract the $\frac{1}{2}$ from $\frac{3}{4}$ which leaves $\frac{2}{4}$ or $\frac{1}{2}$, and the *whole number* being left entire the answer will be $4\frac{1}{2}$.

In some cases, such as this, the fractions may require to be expanded before they can be subtracted, thus :—

$$\begin{aligned} 4\frac{1}{2} - \frac{1}{3} \\ 4\frac{3}{6} - \frac{2}{6} = 4\frac{1}{3}. \text{ Ans.} \end{aligned}$$

In other cases there may be a whole number connected with the subtrahend as well as the minuend, and this will have to be subtracted simply as a whole number, thus :—

$$\begin{aligned} 3\frac{1}{2} - 1\frac{1}{2} \\ 3\frac{4}{4} - 1\frac{2}{4} = 2\frac{2}{4}. \text{ Ans.} \end{aligned}$$

When this is fully explained, ascertain that all understand it by asking a great many such questions as the following :—

QUESTIONS.

How much is $4\frac{1}{2} - \frac{1}{2}$? $3\frac{1}{2} - \frac{1}{2}$? $3\frac{1}{2} - 1\frac{1}{2}$?

$$\begin{array}{l}
 1\frac{1}{2} - \frac{1}{4} ? \quad 7\frac{1}{5} - \frac{1}{5} ? \quad 7\frac{2}{5} - \frac{1}{5} ? \quad 7\frac{2}{5} - \frac{1}{5} ? \\
 7\frac{2}{5} - \frac{1}{5} ? \quad 8\frac{1}{4} - \frac{1}{4} ? \quad 8\frac{2}{4} - \frac{1}{4} ? \quad 8\frac{2}{4} - \frac{1}{4} ? \quad \frac{2}{4} - \frac{1}{4} ?
 \end{array}$$

EXAMPLES.

61. Subtract $3\frac{1}{5}$ from $10\frac{1}{2}$.
62. " $5\frac{1}{2}$ " $8\frac{1}{2}$.
63. " $\frac{2}{3}$ of $\frac{1}{3}$ " $9\frac{2}{3}$.
64. " $1\frac{1}{7}$ " $3\frac{2}{7}$.
65. From $18\frac{1}{2}$ take $7\frac{1}{2}$.
66. " $19\frac{3}{4}$ " $\frac{1}{4}$ of $\frac{1}{8}$.
67. " $44\frac{3}{7}$ " $\frac{1}{3}$ of $\frac{2}{7}$.
68. " $4\frac{1}{4}$ " $4\frac{1}{8}$.
69. Take $\frac{2}{3}$ of $\frac{2}{3}$ from $8\frac{1}{2}$.
70. What is the value of $9\frac{1}{2} - \frac{1}{4}$ of $\frac{1}{5}$?
71. " " $8\frac{1}{2} - \frac{2}{3}$ of $\frac{1}{5}$?
72. " " $9\frac{1}{5} - 9\frac{7}{5}$?
73. " " $8\frac{1}{5} - \frac{1}{2}$ of $\frac{1}{7}$?
74. " " $5\frac{1}{3} - \frac{1}{3}$ of $\frac{1}{2}$?
75. " " $8\frac{1}{2} - \frac{7}{5}$ of $\frac{1}{4}$?
76. " " $3\frac{1}{4} - \frac{1}{4}$ of $\frac{1}{3}$?
77. " " $4\frac{1}{5} - 2\frac{1}{5}$?
78. " " $7\frac{1}{5} - 3\frac{7}{4}$?
79. " " $8\frac{1}{2} - 4\frac{1}{2}$?
80. " " $1894\frac{1}{2} - \frac{1}{4}$ of $\frac{1}{4}$ of $\frac{7}{5}$?

LESSON V.

Sometimes it will happen that the fraction of the subtrahend is greater than the fraction of the minuend. Thus, if we are required to subtract $3\frac{2}{3}$ from $7\frac{1}{3}$, though we can take the 3 from the 7, we cannot take $\frac{2}{3}$ from $\frac{1}{3}$. In this case we must *borrow*, as it is called.

one of the units in 7, so as instead of $7\frac{1}{4}$ we write $6\frac{4}{4}$, which is the same in value. The class may not see this at once, but by giving several examples illustrating it upon the blackboard, they will soon perceive that such a change is only an alteration of form, and that it enables us to subtract without difficulty; thus:—

$$\begin{array}{r} 7\frac{1}{4} - 3\frac{3}{4} \\ 6\frac{4}{4} - 3\frac{3}{4} = 3\frac{1}{4} \text{ Ans.} \end{array}$$

And in the same manner, if the fractions have not a common denominator, so that they require expanding, thus:—

$$\begin{array}{r} 52\frac{1}{8} - 12\frac{3}{8} \\ 51\frac{2}{8} - 12\frac{3}{8} \text{ com. mult. 80.} \\ 51\frac{20}{80} - 12\frac{30}{80} = 39\frac{50}{80} = 39\frac{5}{8} \text{ Ans.} \end{array}$$

EXAMPLES.

81. Subtract $16\frac{3}{8}$ from $17\frac{1}{8}$.
82. " $9\frac{3}{4}$ " $12\frac{1}{2}$.
83. " $8\frac{1}{4}$ " $9\frac{3}{4}$.
84. " $1\frac{1}{2}$ " 20.*
85. From $4\frac{1}{2}$ take $\frac{1}{2}$ of $\frac{1}{2}$.
86. " $5\frac{1}{4}$ " $\frac{1}{2}$ of $\frac{1}{4}$.
87. " $8\frac{2}{8}$ " $7\frac{1}{2}$.
88. " $3\frac{1}{2}$ " $\frac{1}{2}$ of $\frac{1}{2}$.
89. " $8\frac{1}{4}$ " $\frac{1}{4}$.
90. " $5\frac{1}{2}$ " $3\frac{3}{4}$.

LESSON VI.

Sometimes two fractional expressions are so complicated in their

* Of course the number 20 must be changed to the form $19\frac{1}{2}$.

character, that we cannot readily tell which is the largest in value until they are reduced to a common denominator. The difference in value, then, always becomes obvious. For example, if we are asked what is the difference between $\frac{2}{3}$ of $\frac{7}{16}$ and $\frac{3}{4}$ of $\frac{2}{3}$, it would require a very practiced arithmetician to perceive at once which of the two expressions denotes a quantity of the greatest value. But on reducing them thus—

$$\begin{array}{r} \frac{2}{3} \text{ of } \frac{7}{16} \qquad \frac{3}{4} \text{ of } \frac{2}{3} \\ \frac{2 \times 7}{3 \times 16} \qquad \frac{3 \times 2}{4 \times 3} \\ 2)90 \quad 40 \\ \hline 5)45 \quad 20 \\ \hline 9 \quad 4 \end{array}$$

$$2 \times 5 \times 9 \times 4 = 360 \text{ com. den.}$$

Expanding, we have $\frac{140}{360}$, $\frac{80}{360}$.

Thus, the value of the first expression becomes $\frac{140}{360}$, and the second, $\frac{80}{360}$, the first being the greatest by $\frac{60}{360}$. This being fully explained, the class may perform the following examples.

EXAMPLES.

91. What is the difference between $\frac{2}{3}$ of $\frac{7}{16}$ and $\frac{3}{4}$ of $\frac{2}{3}$?
92. Between $6\frac{1}{4}$ and $\frac{2}{16}$ of $\frac{4}{5}$?
93. “ $\frac{3}{16}$ of $\frac{1}{2}$ and $\frac{1}{4}$ of $\frac{1}{2}$?
94. “ $\frac{2}{3}$ of $\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{2}{3}$?
95. “ $3\frac{1}{4}$ and $\frac{3}{16}$ of $\frac{1}{2}$?
96. “ $\frac{1}{2}$ of $\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{1}{16}$?
97. “ $\frac{2}{3}$ of $\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{1}{2}$?
98. “ $\frac{2}{3}$ of $\frac{1}{2}$ and $4\frac{1}{2}$?
99. “ $\frac{2}{3}$ of $\frac{1}{16}$ and $\frac{7}{8}$ of $\frac{1}{16}$?
100. “ $\frac{1}{4}$ of $\frac{2}{3}$ and $\frac{2}{3}$ of $\frac{1}{4}$?
101. “ $1\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{1}{4}$?

LESSON VII.

When the class have acquired a good degree of facility in performing the addition and subtraction of fractions by the ordinary processes, as heretofore described, they may be taught that these processes may often be very much abbreviated thus :—

If several fractions are to be added, it will often happen that some of them will have such forms that the amount can be easily perceived by inspection. For instance, if the pupil has $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{4}$ to add, it is obvious that the amount is $1\frac{1}{2}$. In such a case it is unnecessary to go through the form of expanding and adding the fractions regularly. So when fractions which can easily be added are interspersed among others of a different character, the former may be added separately, and their sum, together with the other fractions which remain, can be written in a line below, and then all added ; thus :—

$$\begin{array}{r} \frac{1}{2} + 2\frac{3}{4} + \frac{1}{8} + \frac{1}{2} + \frac{1}{4} \\ 2\frac{3}{4} + \frac{1}{8} + \frac{3}{4} \end{array}$$

It is obvious that the amounts in the second line are equal to those in the first, and can be now easily added, with much less figuring than would be necessary to expand and add the upper line in the regular manner. So in the following case :

$$\begin{array}{r} 2\frac{1}{3} + 3\frac{1}{4} + \frac{1}{2} + \frac{1}{4} \\ 5\frac{1}{3} + 1 = 6\frac{1}{3} \end{array}$$

The teacher can arrange other examples of this kind, such that the results can be ascertained by a brief mental calculation, and then assign the following examples :

EXAMPLES.

102. Add $\frac{1}{2}$, $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$.
 103. " $\frac{1}{2}$, $8\frac{1}{8}$, $\frac{3}{4}$, $\frac{1}{8}$ and $\frac{7}{8}$.
 " $4\frac{1}{2}$, 4 , $\frac{3}{4}$, $\frac{1}{4}$ and $\frac{1}{4}$.
 5*

105. “ $\frac{1}{2}$ of $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{2}$ and 17.
 106. “ $\frac{3}{7}$, $10\frac{1}{7}$, $\frac{1}{2}$, $\frac{1}{20}$, $\frac{3}{7}$ and 1.

What is the value of the following expressions?

107. $7 - 6\frac{1}{2} + \frac{3}{4} + \frac{3}{4}$.
 108. $8\frac{1}{3} + \frac{2}{3} - 9 + \frac{7}{21} + \frac{1}{3}$.
 109. $3 + \frac{1}{7}$ of $\frac{1}{3} + \frac{1}{21} - 3\frac{1}{2}$.
 110. $18 + \frac{1}{20} + \frac{4}{5} + \frac{3}{10} - \frac{1}{20}$.
 111. $14 + 3\frac{1}{2} - \frac{1}{7} + \frac{1}{7} - 3\frac{1}{2}$.
 112. $4\frac{1}{4} + 9 - \frac{2}{3} + \frac{5}{4} + \frac{3}{4}$.
 113. $16\frac{1}{2} + \frac{3}{4} + 1\frac{1}{2} + 5\frac{1}{2}$.
 114. $\frac{3}{4} + \frac{1}{5} + \frac{3}{5} + 1\frac{1}{5} - \frac{1}{5}$.
 115. $\frac{7}{10} + \frac{1}{5} + \frac{3}{5} - \frac{1}{5} + \frac{4}{15}$.
 116. $3\frac{1}{7} + \frac{7}{9} + \frac{6}{7} + \frac{2}{9} + 1$.
 117. $3\frac{1}{3} + \frac{1}{4}$ of $\frac{1}{2} + \frac{1}{20} + 16\frac{1}{2}$.
 118. $20\frac{1}{2} - 19\frac{1}{2} + 1$.
 119. $34 - 33 + \frac{1}{2}$ of $\frac{1}{4} + 4\frac{3}{4}$.
 120. $34\frac{9}{7} + \frac{1}{7} + 5$.
 121. $1\frac{1}{2} + \frac{1}{4} + 3 + \frac{1}{4}$.

MISCELLANEOUS EXAMPLES.

1. A lady bought a remnant of cloth. It contained $7\frac{1}{4}$ yards. She cut off $\frac{7}{8}$ of a yard for a particular use; how much remained?

2. Says Mr. A. to Mr. B., My brother owned $\frac{4}{5}$ of the ship Baltic, and I bought one-third of his share. Says Mr. B. to Mr. A., My brother owned the other fifth, and I bought the whole of his share. How much more do you own than I? What should have been his answer?

3. Two boys were reading books. William's book had 375 pages, and he had read 150 of them. George's contained 360, and he had read 140. Which had read the greatest portion of his book ?

4. A ship-carpenter, a sail-maker, and a merchant, entered into a partnership to build a ship. The carpenter built the hull, which was $\frac{2}{3}$ of the whole value. The sail-maker furnished the rest, and the merchant paid to the carpenter one-fifth, and to the sail-maker one-quarter of what they had respectively furnished. What then was each one's share of the ship ?

5. Three boys built a dam. James was to own one-half, and George and William one-quarter each. Afterwards Samuel bought one-half of James's share, and one-quarter of George's. How much did James have left ?

6. What remains if you take $\frac{1}{2}$ of $\frac{2}{3}$ from $\frac{7}{16}$ of $\frac{1}{4}$?

7. A boy had 20 feet of twine. He gave away ten feet and three-quarters of a foot. How much had he left ?

8. Suppose he had given away 3 feet and 2 inches, how much would he have had left in feet and parts of a foot, an inch being $\frac{1}{12}$ of a foot ?

9. Two farmers had each an orchard. A.'s orchard was $\frac{1}{5}$ pear-trees, and $\frac{4}{5}$ apple-trees. B.'s orchard was $\frac{1}{3}$ pear-trees and $\frac{2}{3}$ apple-trees. An insect called a borer, attacked the apple-trees in both orchards and destroyed $\frac{1}{4}$ of A.'s and $\frac{1}{3}$ of B.'s. What proportion of each of the two orchards remained ?

10. Mr. A. and Mr. B. owned shares in a ship. Mr. A. said, I own $\frac{5}{8}$ and intend giving $\frac{1}{4}$ of my

share to my son. Mr. B. said, I own $\frac{1}{4}$ and intend giving $\frac{1}{3}$ of my share to my son. If this were done, which son would own the largest share, and how much the largest?

11. Two travelers set out upon a pedestrian tour among the Alps. After walking some miles from their hotel, they came to a village. They passed the village, and came pretty soon to a guide-post, which said it was $\frac{3}{4}$ of a league back to the village, and $7\frac{1}{4}$ leagues back to the hotel. How far was it from the hotel to the village?

12. Some travelers had a journey of fourteen miles and three-quarters to take. They walked on till they had gone seven miles, and seven-eighths of a mile, How much of their journey remained?

13. An upholsterer measured the rooms of a house for the purpose of carpeting it, and found it would require in all $102\frac{3}{4}$ yards. The owner afterwards concluded to omit one of the rooms, which would have required $23\frac{1}{4}$ yards. How much would be required for the rest?

V. MULTIPLICATION OF FRACTIONS.

LESSON I.

It is very important that the pupil should obtain a clear idea at the outset of what multiplication by a fraction is. To assist in this, ask such questions as the following :—

What is multiplying a number by 10 ? The pupils will be likely to answer—It is taking it ten times. What is multiplying a number by 5 ? *Ans.* Taking it 5 times. What is multiplying a number by 3 ? *Ans.* Taking it 3 times. What is multiplying a number by 1 ? *Ans.* Taking it once. And what is multiplying a number by $\frac{1}{2}$? *Ans.* Taking it — The pupils will perhaps be at a loss for an expression, but they will be inclined to say something like—Taking it half a time.

The teacher will tell them that the idea intended by that language is correct, though the language itself is awkward, and then he may say—

We will begin again, and use a little different phraseology. What is multiplying any number, as 6 for example, by 10 ? Answer in this form. It is taking *ten* sixes. *Class* (repeating the answer given them.) It is taking *ten* sixes. What is multiplying 6 by 2 ? *Class.* It is taking *two* sixes. What is multiplying 6 by 1 ? *Class.* It is taking *one* six. And what then is multiplying 6 by $\frac{1}{2}$? *Class.* It is taking *one-half* of six.

In such ways as this, varied according to the ingenuity of the teacher, the pupils may be led to *generalize* their idea of multiplication, and see why multiplication by a fraction diminishes the number multiplied, although it is very difficult to give a general definition in language scientifically correct.

Show to the pupil that since multiplying any number by a fraction as, for example, $\frac{3}{4}$, is, as has been above explained, obtaining $\frac{3}{4}$ of the number, the process of multiplying one fraction by another becomes the same with reducing a compound fraction to a simple one, as before explained. For let us suppose that we wish to multiply $\frac{4}{7}$ by $\frac{3}{4}$. It is only to obtain $\frac{3}{7}$ of $\frac{4}{7}$, which is simply reducing a compound fraction to a simple one. This is done by multiplying the numerators together for a new numerator, and the denominators together for a new denominator. This is, therefore, the process in multiplication of fractions. In the following example, the result should be reduced to its lowest terms.

EXAMPLES.

- | | |
|---|---|
| 1. $\frac{3}{4} \times \frac{5}{6}$. | 11. $\frac{4}{5} \times \frac{1}{18}$. |
| 2. $\frac{4}{5} \times \frac{3}{4}$. | 12. $\frac{3}{5} \times \frac{4}{5}$. |
| 3. $\frac{1}{7} \times \frac{2}{30}$. | 13. $\frac{2}{100} \times \frac{4}{1200}$. |
| 4. $\frac{1}{30} \times \frac{1}{20}$. | 14. $\frac{4}{5} \times \frac{7}{20}$. |
| 5. $\frac{1}{6} \times \frac{3}{4}$. | 15. $\frac{1}{3} \times \frac{5}{30}$. |
| 6. $\frac{3}{4} \times \frac{4}{5}$. | 16. $\frac{6}{12} \times \frac{4}{30}$. |
| 7. $\frac{1}{15} \times \frac{1}{20}$. | 17. $\frac{3}{5} \times \frac{1}{6}$. |
| 8. $\frac{3}{4} \times \frac{1}{5}$. | 18. $\frac{4}{5} \times \frac{1}{3}$. |
| 9. $\frac{1}{4} \times \frac{4}{5}$. | 19. $\frac{1}{4} \times \frac{1}{4}$. |
| 10. $\frac{1}{4} \times \frac{3}{10}$. | 20. $\frac{1}{4} \times \frac{1}{2}$. |

LESSON II.

In all operations with fractions, whole numbers are of the nature of numerators. Thus, 5 is five *ones*, or as it would be written in a fractional form, $\frac{5}{1}$. Whole numbers may, therefore, be written in that form, and then managed like fractions.

For instance, suppose it to be required to multiply $\frac{3}{4}$ by 7. We can perform the work thus :—

$$\frac{3}{8} \times 7$$

$$\frac{3}{8} \times \frac{7}{1} = \frac{21}{8} = 3\frac{3}{8} = 3\frac{1}{2}. \text{ Ans.}$$

EXAMPLES.

- | | |
|---|--|
| 21. $\frac{4}{5} \times 3.$ | 31. $\frac{3}{7} \times 1849.$ |
| 22. $\frac{4}{7} \times 10.$ | 32. $\frac{22}{5} \times \frac{7}{15}.$ |
| 23. $1 \times \frac{5}{25}.$ | 33. $\frac{8}{9} \times \frac{1}{14}.$ |
| 24. $\frac{3}{15} \times \frac{1}{2}.$ | 34. $\frac{3}{4} \times 5.$ |
| 25. $\frac{7}{9} \times 184.$ | 35. $\frac{7}{15} \times 10.$ |
| 26. $\frac{3}{4} \times \frac{8}{9} \times 14.$ | 36. $\frac{3}{8} \times 9.$ |
| 27. $\frac{1}{3} \times \frac{1}{7}.$ | 37. $\frac{4}{5} \times 5.$ |
| 28. $25 \times \frac{7}{15}.$ | 38. $\frac{3}{4} \times \frac{3}{8} \times 460.$ |
| 29. $\frac{8}{9} \times 4.$ | 39. $\frac{3}{14} \times \frac{1}{5}.$ |
| 30. $\frac{1}{8} \times \frac{9}{15}.$ | 40. $\frac{3}{4} \times 4.$ |

LESSON III.

The pupils having multiplied the above examples by converting every whole number into a fraction, may now be taught that, as in so doing, they make the denominator always 1, and as multiplying by 1 makes no change in the value, the operation in all cases where a fraction is to be multiplied, resolves itself simply into multiplying the numerator by the whole number, and writing the denominator under it, just as it was before, and then reducing the fraction to its lowest terms. And as in reducing an improper fraction to a mixed number we divide by the denominator, the whole operation becomes multiplying the whole number by the numerator of the fraction, and dividing the product by the denominator, thus :—

$$\frac{3}{8} \times 4$$

$$\frac{3}{8} \times \frac{4}{1} = \frac{12}{8} = 2\frac{1}{2}. \text{ Ans.}$$

This is the same as

$$\frac{3}{5} \times 4 = \overline{3 \times 4} \div 5 = 2\frac{2}{5}. \text{ Ans.}$$

This being fully explained and illustrated by several examples upon the blackboard, the following examples may be performed by the class at their seats. Of course, when more than two fractions are given to be multiplied together, the first and second may be multiplied as above, and that product by the third. Or the three numerators may be multiplied together, and then the three denominators, which will give the new fraction.

EXAMPLES.

41. Multiply $\frac{3}{5}$ by 5.
42. " $\frac{4}{5}$, 21, and $\frac{1}{10}$ together.
43. " $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$ together.
44. What is ten times $\frac{1}{10}$?
45. What is $\frac{1}{2}$ of $\frac{3}{4}$?
46. What is the product of $\frac{7}{8}$, $\frac{3}{4}$, and 3?
47. " " $\frac{5}{6}$, $\frac{7}{8}$, and 5?
48. Required the product of $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{3}{8}$.
49. Multiply $\frac{7}{12}$ by 84.
50. What is $\frac{1}{3}$ of 100?
51. " $\frac{7}{8}$ of 180?
52. " $\frac{3}{4}$ of $\frac{1}{2}$ of 6?
53. Multiply $\frac{3}{5}$ by $\frac{7}{8}$ of 18.
54. " $\frac{1}{2}$ by 33.
55. " $\frac{1}{3}$ by 79.
56. Required the product of $\frac{7}{12}$ and 18.
57. What is the product of $\frac{1}{2}$ and $\frac{3}{4}$?
58. Multiply $\frac{3}{5}$ by $\frac{7}{8}$ of 18.
59. What is the product of $\frac{3}{4}$ and $\frac{1}{2}$?
60. Multiply $\frac{1}{3}$ by 4671.

LESSON IV.

The pupils will be very prone to confound the process of multiplying fractions with that of reducing them to a common denominator, which is done in preparation for adding them. The distinction must be carefully pointed out to them. In reducing fractions to a common denominator, we *expand* them, that is, we multiply both numerator and denominator by the same number, which does not alter their value. For example, in reducing $\frac{3}{4}$ and $\frac{1}{5}$ to a common denominator, we expand $\frac{3}{4}$ by multiplying both the 3 and the 4 by 5, and we expand $\frac{1}{5}$ by multiplying both the 1 and the 5 by 4; thus obtaining in the place of the original fractions, $\frac{15}{20}$ and $\frac{4}{20}$; expressions which, though new in form, denote the same value as before. But in *multiplying* $\frac{3}{4}$ by $\frac{1}{5}$, we multiply the 3 by the 1, and the 4 by the 5, thus obtaining in the place of the two original fractions, *one* expression which is not equal in value to either of them, but the product of both. Multiplying two fractions is, however, the same as reducing a compound fraction to a simple one.

EXAMPLES.

61. Reduce $\frac{4}{5}$ and $\frac{3}{8}$ to a common denominator.
62. Multiply $\frac{4}{5}$ by $\frac{3}{8}$.
63. Reduce $\frac{4}{5}$ of $\frac{3}{8}$ to a simple fraction.
64. Reduce $\frac{3}{8}$ and 3 to a common denominator.
65. Multiply $\frac{3}{8}$ by 3.
66. What is the value of $\frac{3}{8}$ of $\frac{3}{8}$ of 5?
67. " " " $\frac{3}{8} + \frac{3}{8}$?
68. What is the value of $36 \times \frac{3}{8}$ of 8?
69. Reduce $\frac{7}{8}$, $\frac{3}{8}$ and $\frac{1}{8}$ to a common denominator.
70. Multiply $\frac{7}{8}$, $\frac{3}{8}$ and $\frac{1}{8}$.
71. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{8}$.
72. Multiply $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{8}$ together.

73. What is $\frac{3}{4}$ of $\frac{7}{10}$?
 74. Multiply 360 by $\frac{1}{4}$.
 75. " 420 by $\frac{1}{2}$ of $\frac{1}{3}$.
 76. " 12 by $\frac{1}{4}$ of 13.
 77. " 45 by $\frac{6}{15}$.
 78. " 24 by $3 \times \frac{2}{3}$.
 79. " 2 by $\frac{1}{4} \frac{3}{4}$.
 80. Reduce $\frac{3}{4}$ and $\frac{1}{5}$ to a common denominator.

LESSON V.

When mixed numbers or compound fractions are given to be multiplied, the work can easily be performed by first reducing the former to improper fractions, and the latter to simple ones. The best way to arrange the work in this case is to write the expressions to be multiplied in a line, in their original forms, and then to reduce them one by one, and write them again, in their new forms, in a line below, taking care to place each new expression under its equivalent, thus:—

$$\begin{array}{l} \frac{3}{4} \text{ of } \frac{1}{2} \times \frac{7}{10} \times 4\frac{1}{2}. \\ \frac{3}{4} \times \frac{1}{2} \times \frac{7}{10} \times \frac{13}{2} = \frac{273}{40} = 6\frac{33}{40}. \text{ Ans.} \end{array}$$

If there are any fractions given which may be reduced to lower terms, this may be done before multiplying, and it will simplify the process. For as reducing the fraction to simple terms does not alter the value, it is plain it cannot change the result, thus :

$$\begin{array}{l} \frac{1}{10} \times \frac{1}{2} \times \frac{1}{10}. \\ \frac{1}{2} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{2} = \frac{1}{2}. \text{ Ans.} \end{array}$$

EXAMPLES.

81. Multiply together $7\frac{1}{2}$, $\frac{3}{4}$ and 4.

82. Multiply together $1, \frac{2}{3}$ and $\frac{1}{2}$.
83. " " $\frac{7}{10}$ of 6 and $\frac{1}{2}$ of 18.
84. What is the product of 8, $\frac{9}{10}$ and $3\frac{1}{2}$?
85. " " $3\frac{1}{2}, \frac{7}{10}$ and $\frac{2}{3}$?
86. " " $3\frac{1}{2}, 3\frac{1}{2}$ and $3\frac{1}{2}$?
87. " " $\frac{1}{2}$ of $3\frac{1}{2}$ and $\frac{7}{10}$?
88. " " $\frac{4}{5}$ and $\frac{1}{5}$?
89. " " $\frac{2}{3}$ of 16 and $\frac{1}{2}$ of 10?
90. " " $8\frac{1}{2} \times \frac{2}{3}$ of 10?
91. " " $9\frac{1}{10} \times \frac{1}{10}$?
92. " " $\frac{1}{10} \times \frac{2}{100} \times \frac{1}{100}$?
93. " " $\frac{2}{3} \times 1\frac{5}{10}$?
94. " " $6\frac{1}{2} \times 3\frac{1}{2}$?
95. " " $5 \times \frac{1}{2}$ of $2\frac{3}{4}$?
96. " " $8 \times 7\frac{1}{2}$?
97. " " $3\frac{1}{2} \times 4\frac{1}{2}$?
98. " " $5\frac{1}{2} \times 7\frac{1}{2}$?
99. " " $326\frac{1}{2} \times 425\frac{1}{2}$?
100. " " $\frac{1}{2} \times 7\frac{1}{10}$?

LESSON VI.

When the whole number is large, in such cases as those near the close of the list of examples in the last lesson, it is sometimes more convenient to take a different course from that of changing the mixed numbers to improper fractions. If, for instance, we have $184\frac{2}{3}$ to be multiplied by $720\frac{1}{2}$, we can do it without changing the numbers into improper fractions, by multiplying each part of the new multiplicand by each part of the multiplier, and then adding four products thus obtained. The teacher should perform the operation upon the blackboard for the class, in the following manner:

$$\begin{array}{r}
 6\frac{2}{3} \\
 30\frac{1}{2} \\
 \hline
 \end{array}$$

$\frac{2}{3}$ product of $\frac{2}{3}$ by $\frac{1}{2}$.
 3 " 6 " $\frac{1}{2}$.
 12 " $\frac{2}{3}$ " 30.
 180 " 6 " 36.

$$195\frac{2}{3} = 195\frac{1}{2}. \text{ Ans.}$$

Sometimes several of these partial products will contain fractions, as in the following case, where $4\frac{1}{2}$ is to be multiplied by $3\frac{2}{3}$.

$$\begin{array}{r}
 4\frac{1}{2} \\
 3\frac{2}{3} \\
 \hline
 \end{array}$$

$\frac{2}{3}$ product of $\frac{2}{3}$ by $\frac{1}{2}$.
 $\frac{1}{2}$ " $\frac{2}{3}$ " 4.
 $\frac{2}{3}$ " 3 " $\frac{1}{2}$.
 12 " 3 " 4.

Then adding the above products, we have

$$\begin{aligned}
 &12 + \frac{2}{3} + \frac{1}{2} + \frac{2}{3}. \\
 &12 + \frac{2}{3} + \frac{2}{3} + \frac{1}{2} = 12\frac{4}{3} = 14\frac{1}{3}. \text{ Ans.}
 \end{aligned}$$

Explain to the pupil that when the numbers are quite large, the process becomes much longer, though it is performed in the same manner. They, will in such cases, find it convenient to arrange the work in the following manner :

Work upon the Slate.

$$(101.) \quad 84\frac{1}{2} \times 25\frac{2}{3}.$$

$$\begin{aligned}
 \frac{3}{8} \times \frac{1}{7} &= \frac{3}{56} \\
 \frac{3}{8} \times 84 &= 28 \\
 25 \times \frac{1}{7} &= \frac{25}{7} = 3\frac{4}{7} \\
 25 \times 84 &= 2100 \\
 \hline
 2131\frac{4}{7} &= 2131\frac{4}{7}. \text{ Ans.}
 \end{aligned}$$

EXAMPLES.

101. Multiply $84\frac{1}{7}$ by $25\frac{3}{4}$.
102. " $320\frac{1}{8}$ " $3\frac{1}{2}$.
103. " $145\frac{3}{4}$ " $19\frac{1}{18}$.
104. " $4\frac{1}{2}$ " $3\frac{3}{4}$.
105. " $5\frac{1}{7}$ " $18\frac{1}{2}$.
106. " $51\frac{1}{18}$ " $3\frac{1}{3}$.
107. " $4\frac{1}{8}$ " $5\frac{1}{2}$.
108. " $4\frac{1}{9}$ " $19\frac{2}{9}$.
109. " $5\frac{1}{8}$ " $17\frac{3}{8}$.
110. " $10\frac{1}{18}$ " $1\frac{1}{18}$.

VI. DIVISION OF FRACTIONS.

LESSON I.

EXPLAIN to the class that division by a fraction is very easily performed, inasmuch as all that is necessary is to invert the divisor, that is, to change the places of the numerator and denominator, and then it becomes a case of multiplication. Thus, if we wish to divide $\frac{2}{3}$ by $\frac{1}{2}$, we say $\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3} = 1\frac{1}{3}$. That is, $\frac{1}{2}$ will go in $\frac{2}{3}$ *one and one-fifth* times.

The work may be arranged as above upon the slate, or it may be written in two lines, thus :—

$$\begin{array}{l} \frac{2}{3} \div \frac{1}{2}. \\ \frac{2}{3} \times \frac{2}{1} = \frac{4}{3} = 1\frac{1}{3}. \text{ Ans.} \end{array}$$

In either case all that is to be done is to change the places of the numerator and denominator *in the divisor*, and place the sign of multiplication between. The pupils must remember that it is the *divisor* which must be thus inverted, and not the dividend. The answers are to be reduced to the lowest terms.

EXAMPLES.

1. Divide $\frac{4}{5}$ by $\frac{2}{3}$.
2. “ $\frac{3}{8}$ “ $\frac{1}{3}$.
3. “ $\frac{1}{6}$ “ $\frac{1}{8}$.
4. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$?

5. How many times is $\frac{1}{3}$ contained in $\frac{1}{3}$?
6. " " $\frac{1}{15}$ " $\frac{1}{3}$?
7. Multiply 4 by $\frac{1}{3}$ and then divide by $\frac{1}{15}$.
8. Divide $\frac{7}{15}$ by $\frac{2}{3}$ of $\frac{1}{3}$.
9. " $\frac{1}{3}$ " $\frac{2}{15}$.
10. What is the value of $\frac{1}{3}$ of $\frac{1}{3}$?
11. " " $\frac{1}{3} \div \frac{1}{3}$?
12. " " $\frac{1}{3} \times \frac{1}{3}$?
13. Divide $\frac{4}{15}$ by $\frac{2}{3}$ and then multiply the quotient by $\frac{1}{3}$.
14. What is the value of $\frac{1}{3} \times \frac{2}{3}$ of $\frac{1}{3}$?
15. " " $\frac{2}{15} \div \frac{1}{3}$?
16. Divide $\frac{1}{3}$ by $\frac{1}{3}$.
17. " $\frac{1}{3}$ by $\frac{1}{3}$.
18. " $\frac{2}{3}$ by $\frac{1}{3}$.
19. Multiply $\frac{1}{3}$ by $\frac{1}{3}$.
20. " $\frac{1}{3}$ by $\frac{1}{3}$.

LESSON II.

The pupils may easily be led to see that if the denominators of two fractions are alike, it will be sufficient to divide one numerator by the other. Thus, if we are required to determine how many times $\frac{3}{10}$ are contained in $\frac{6}{10}$, it is obvious that the answer is, *Twice*, for 6 will contain 3 twice, whether the six and three are tenths or anything else.

This may also be shown to them in accordance with the method of procedure explained in the last lesson, thus: If we have the above two fractions given, and are to divide one of them by the other, we invert the last, and then multiply, thus:—

$$\frac{6}{10} \div \frac{3}{10} = \frac{6}{10} \times \frac{10}{3}.$$

Now it is plain that if we multiply the two numerators together, we get ten times six, and the two denominators give ten times three. The answer would then be $\frac{60}{30}$, and to reduce this to its lowest terms, we may divide both by 10 again, giving $\frac{6}{3}$, which becomes 2. Thus, if we multiply the two terms by 10, it is only to prepare the way to divide them again by the same number. In the same manner in this instance :

$$\frac{1}{4} \div \frac{2}{4} = \frac{1}{4} \times \frac{4}{2} = \frac{4}{8} = \frac{1}{2}.$$

Here, if we multiply by the 4, it is only to divide again by it immediately afterwards. We may, therefore, safely omit to include the denominator in such case, and merely say, 1, the first numerator, divided by 2, the second numerator, gives $\frac{1}{2}$ the answer.

It will require several illustrations to make this familiar to the pupils. When it is well understood, they may be told that when the two denominators are unlike, they can, if they choose, reduce the fractions to a common denominator, and then merely divide the numerators ; thus :—

$$\begin{aligned} 2\frac{1}{3} \div 1\frac{1}{6} \\ 4\frac{2}{3} \div 1\frac{1}{6} = 4\frac{2}{1} = 30. \text{ Ans.} \end{aligned}$$

EXAMPLES.

21. How many times is $\frac{2}{7}$ contained in $\frac{3}{7}$?

22. “ “ $1\frac{2}{3}$ “ $1\frac{1}{3}$?

23. “ “ $1\frac{2}{3}$ “ $1\frac{1}{3}$?

24. Divide $1\frac{1}{3}$ by $1\frac{7}{8}$.

25. “ $1\frac{1}{3}$ by $1\frac{7}{8}$.

Of course, in such a case as this, the answer obtained by dividing 16 by 7 is 2 $\frac{2}{7}$.

26. Divide $1\frac{1}{3}$ by $1\frac{7}{8}$.

27. “ $1\frac{1}{3}$ by $1\frac{7}{8}$.

28. Divide $\frac{4}{3}$ by $\frac{1}{3}$.
 29. " $\frac{3}{5}$ by $\frac{2}{5}$.
 30. " $\frac{2}{3}$ by $\frac{1}{3}$.

Expand the second fraction by 2, and then divide one numerator by the other.

31. Divide $\frac{1}{4}$ by $\frac{3}{4}$.
 32. " $\frac{1}{4}$ by $\frac{3}{8}$.
 33. " $\frac{1}{3}$ by $\frac{1}{6}$.
 34. " $\frac{1}{4}$ by $\frac{1}{8}$.

Of course, in this case the answer obtained by dividing 18 by 19 will be $\frac{18}{19}$.

35. Divide $\frac{1}{7}$ by $\frac{1}{7}$.
 36. " $\frac{1}{2}$ by $\frac{3}{4}$.
 37. " $\frac{7}{10}$ by $\frac{1}{10}$.
 38. " $\frac{4}{6}$ by $\frac{2}{3}$.
 39. " $\frac{3}{8}$ by $\frac{4}{8}$.
 40. " $\frac{1}{18}$ by $\frac{1}{18}$.

LESSON III.

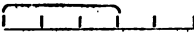
Explain to the pupil that when a fraction is to be divided by a whole number, we either divide the numerator, or multiply the denominator. This has been already shown in the explanations given in relation to the general nature of fractions. It would be well to turn back to Lesson IX., page 41, and recall those processes to the mind of the pupil. It is also desirable to explain as follows the reasons for this.

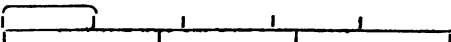
The propriety of dividing the numerator, when it can be divided, is very obvious. Thus $\frac{4}{3}$ divided into 4 parts will obviously give 1

for each part, and $\frac{1}{4}$ is obtained from $\frac{4}{4}$ by dividing 4, the numerator, by 4, and preserving the denominator unchanged.

In respect to multiplying the denominator of a fraction as a method of dividing it, the class may be reminded of the explanation given in the lessons above referred to, where it is shown that multiplying the denominator increases the number, and so diminishes the value of the parts into which the unit is divided. But there is another principle which throws great light on the subject, which ought to be distinctly brought before the minds of the pupils. It is this :—

All denominators are in their very nature *divisors*. Thus, $\frac{1}{4}$ is 1 divided by 4, and $\frac{6}{7}$ is 6 divided by 7. For it makes no difference, whether we consider $\frac{6}{7}$ as meaning that one thing is divided into seven parts, and six of them are taken, or that six things are taken and divided into seven parts : that is, one-seventh of six is the same as six-sevenths of one. To see clearly the distinction pointed out by this phraseology will require considerable explanation, and some visible illustrations as follows, using $\frac{3}{4}$ as the fraction :

$\frac{3}{4}$ of one unit 

$\frac{1}{4}$ of three units 

The pupils may be made to see by some illustrations like the above, that $\frac{3}{4}$ may be considered as three units divided into five parts, and three of them taken. In either case the denominator 5 is a divisor. Let the pupils understand that a fraction may always be read on this principle. Thus $\frac{4}{7}$ may be read *four-sevenths*, or *four divided by seven*. In fact, this last mode is altogether the most usual, especially for large fractions, some of which, indeed, can hardly be read at all in any other way. Thus $\frac{4}{163}$ is much better read *4 divided by 163* than in any other manner.

All denominators being thus divisors, when we wish to divide $\frac{6}{7}$ by 3, the case is this : We have 6 already divided by 7, and we are now required to divide the result by 3 again. But we may as well combine these two divisors, and say, 6 divided by 21, as to say,

6 divided by 7, and then by 3. This gives $\frac{6}{21}$ as an expression of the ultimate value. Thus multiplying the denominator of a fraction by a divisor, is only combining two divisors by multiplying them together.

A denominator being thus always of the nature of a divisor, a numerator is always of the nature of a multiplier—unity, or one, being in all cases understood as the quantity multiplied and divided, unless some other number is expressed. For instance, $\frac{1}{6}$ is 1 multiplied by 6 and divided by 7, or it is 1 divided by 7 and then multiplied by 6. So when any other number is expressed. Thus $\frac{20}{5}$ of 20 is 20 multiplied by 4, and then divided by 5, or it is 20 divided by 5 and then multiplied by 4. Whichever way the calculation is made, the result will be the same; as should be shown by making some such calculations on the blackboard. Do this in repeated instances, until it is fully impressed upon the minds of the class, that a numerator is always a multiplier, and a denominator always a divisor.

This being understood, will explain also why, when we wish to multiply a fraction by a number, we multiply the numerator. For the numerator being already a multiplier, and another multiplier being given, we simply combine them by multiplying them together.

So if we wish to multiply two fractions together, we multiply the numerators and also the denominators. This is merely combining the two multipliers and also the two divisors.

When these principles are fully understood, the following examples may be assigned; the pupils being directed in all cases to combine the new divisor with the one already existing in the denominator of the fraction, unless they can divide the numerator.

EXAMPLES.

- | | |
|---------------------------------|---------------------------------|
| 41. Divide $\frac{7}{16}$ by 4. | 47. Divide $\frac{4}{3}$ by 48. |
| 42. " $\frac{1}{2}$ by 167. | 48. " $\frac{1}{2}$ by 2. |
| 43. " $\frac{1}{3}$ by 15. | 49. " $\frac{1}{3}$ by 10. |
| 44. " $\frac{2}{3}$ by 186. | 50. " $\frac{1}{3}$ by 4. |
| 45. " $\frac{1}{3}$ by 5. | 51. " $\frac{4}{7}$ by 3. |
| 46. " $\frac{1}{4}$ by 16. | 52. " $\frac{1}{4}$ by 300. |

- | | |
|---------------------------------|----------------------------------|
| 53. Divide $\frac{1}{18}$ by 4. | 57. Divide $\frac{4}{3}$ by 365. |
| 54. " $\frac{7}{16}$ by 18. | 58. " $\frac{4}{3}$ by 18. |
| 55. " $\frac{1}{8}$ by 18. | 59. " $\frac{7}{18}$ by 19. |
| 56. " $\frac{1}{7}$ by 7. | 60. " $\frac{1}{3}$ by 8. |

LESSON IV.

To divide a whole number by a fraction, we invert the divisor, and then it becomes a case of multiplication. Thus $4 \div \frac{2}{3}$ is $4 \times \frac{3}{2}$, and the process indicated by the last expression is to be performed by multiplying the 4 by 3, which is the multiplier, and dividing by 2, which, being a denominator, is a divisor. This gives us $\frac{12}{2}$, or 6 , for the answer.

EXAMPLES.

- | | |
|---------------------------------|--|
| 61. Divide 8 by $\frac{2}{7}$. | 71. Divide 4 by $\frac{1}{3}$. |
| 62. " 19 by $\frac{1}{5}$. | 72. " 3 by $\frac{1}{4}$. |
| 63. " 356 by $\frac{1}{18}$. | 73. " 5 by $\frac{2}{3}$. |
| 64. " 3 by $\frac{1}{2}$. | 74. " 7 by $\frac{1}{7}$. |
| 65. " 17 by $\frac{1}{17}$. | 75. " 21 by $\frac{1}{3}$. |
| 66. " 4 by $\frac{1}{4}$. | 76. " $1\frac{2}{3}$ by 4. |
| 67. " 78 by $\frac{2}{9}$. | 77. " 2 by $\frac{1}{2}$. |
| 68. " 20 by $\frac{4}{5}$. | 78. " $\frac{1}{2}$ by 2. |
| 69. " 220 by $\frac{1}{2}$. | 79. " $\frac{4}{5}$ by 3. |
| 70. " 2 by $\frac{1}{2}$. | 80. " $\frac{1}{7}$ by $\frac{1}{7}$. |

LESSON V.

In multiplying and dividing fractions, the process can sometimes be much abbreviated by canceling such numerators and denominators as are equal to one another. Let this be explained to the class.

by the aid of a simple illustration like the following. Suppose the following fractions are to be multiplied :

$$\frac{4}{3} \times \frac{5}{9} \times \frac{3}{4}.$$

Here we have 4 as a *numerator* in the first fraction, and a *denominator* in the last. And we have 5 as a numerator in the second fraction, and a denominator in the first. By canceling these, that is, striking them out, and substituting 1 in their places, we have

$$\frac{1}{3} \times \frac{1}{9} \times \frac{3}{1} = \frac{3}{27} = \frac{1}{9}. \text{ Ans.}$$

Whereas, if we had multiplied all the original numerators and denominators together, we should have had for the product $\frac{40}{27}$, which we should have had to reduce to its lowest terms by dividing by 4 and 5 successively. That is, we should multiply by 4 and 5 only for the sake of dividing again. When, therefore, we have any number among the numerators, and *the same* among the denominators, we may cancel them previous to multiplying the fractions.

Explain also to the class cases like this, viz., $\frac{3}{8} \times \frac{16}{17}$, where one of the numerators, 16, is twice as great as one of the denominators, 8. In such a case we cancel the 8 and *one of the factors* of 16, and write 2, the other factor, in its place. The fractions then become $\frac{3}{1} \times \frac{2}{17} = \frac{6}{17}$. *Ans.* The pupils will be in danger of putting down the *remainder* instead of the quotient, in such case, unless they are specially instructed that when they cancel a factor from a numerator or denominator, they must write the *other factor* in its place ; thus :—

$$\begin{array}{l} \frac{3}{8} \times \frac{16}{17} \times \frac{1}{2} \\ \frac{3}{1} \times \frac{2}{17} \times \frac{1}{1} = \frac{6}{17}. \text{ Ans.} \end{array}$$

EXAMPLES.

81. What is the value of $\frac{3}{17} \times \frac{4}{5} \times \frac{17}{4}$?
 82. “ “ $\frac{4}{5} \times \frac{14}{15} \times \frac{15}{14}$?

83. What is the value of $\frac{2}{10} \times \frac{3}{8}$?
 84. " " $\frac{4}{5} \times \frac{1}{2} \times \frac{3}{4}$?
 85. " " $\frac{3}{7} \times \frac{7}{5} \times \frac{2}{10}$?
 86. " " $\frac{5}{6} \times \frac{6}{7}$?
 87. " " $\frac{3}{10} \times \frac{2}{7} \times \frac{7}{5}$?

In the above example the first two denominators together cancel the last numerator.

88. What is the value of $\frac{2}{10}$ of $\frac{1}{10}$ of $\frac{1}{5}$?
 89. " " $\frac{2}{7}$ of $\frac{7}{3}$ of $\frac{1}{4}$?
 90. " " $\frac{2}{7}$ of $\frac{7}{8}$ of $\frac{1}{4}$?
 91. Multiply $\frac{7}{10}$ of $\frac{1}{2}$ by $\frac{1}{5}$ of $\frac{3}{8}$.
 92. " $\frac{1}{2}$ by $\frac{1}{2}$ of $\frac{5}{6}$.
 93. " $\frac{3}{10}$ of 15 by 10.

Whole numbers, it must be remembered, are of the nature of numerators.

94. Multiply $\frac{2}{7}$ by $\frac{3}{8}$.
 95. What is the value of $\frac{1}{10} \times \frac{2}{3}$ of $\frac{4}{5}$?
 96. " " $\frac{1}{2}$ of $\frac{6}{7} \times \frac{7}{10}$ of $\frac{3}{4}$?
 97. " " $\frac{1}{17} \times \frac{3}{5}$?
 98. " " $\frac{2}{3}$ of $\frac{5}{10}$ of $\frac{1}{2}$?
 99. " " $\frac{1}{7}$ of $\frac{2}{3}$ of $\frac{7}{10}$?
 100. " " $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{1}{2}$?

MISCELLANEOUS EXAMPLES.

1. An upholsterer had a piece of carpet $17\frac{1}{2}$ yards long. He wished to find how many stool coverings it would make, each stool cover requiring $\frac{7}{4}$ of a yard.
2. How many whole coverings would there have

been and how much over, if each covering had required $\frac{1}{3}$ of a yard ?

In this case, by reducing the fractions to a common denominator, and dividing one by the other, we get the number of coverings and parts of coverings, for the answer, whereas, we want the number of coverings and parts of a yard left over. The parts of a covering may be converted into parts of a yard by making a compound fraction. Each covering is $\frac{1}{3}$ of a yard ; of course $\frac{1}{3}$ of a covering will be $\frac{1}{9}$ of $\frac{1}{3}$ of a yard.

3. Suppose the coverings were to require $\frac{1}{10}$ of a yard each, and the piece of carpeting had been $18\frac{1}{2}$ yards, how many coverings would there be, and what part of a covering over ?

4. How many coverings and what part of a yard over ?

5. Divide $35\frac{1}{2}$ yards of twine among 7 boys, so as to give each one an equal share.

6. Divide it in such a manner as to give each boy $5\frac{1}{4}$ yards. How many boys would receive a full share, and how much would be left over ?

7. Three boys were carrying a heavy parcel. The distance was $\frac{3}{4}$ of a mile. How large a part of a mile must each one carry it, supposing the road to be divided equally among them ?

8. How many soldiers will 4,000 pounds of bread supply, supposing each one to have $\frac{3}{4}$ of a pound ?

9. A merchant owned $\frac{7}{8}$ of a ship, and he wished to give $\frac{1}{10}$ of it to each of his sons. How many sons could he supply with such a portion ?

10. Multiply $\frac{7}{8}$ by $\frac{9}{10}$ and then divide by $\frac{1}{4}$.

11. Some young men in the woods came to a torrent, and they had to cut a tree down in order to get across it. The stream was $10\frac{1}{2}$ feet wide at the surface of the water, and the width on the top of the bank was once and a half what it was at the water. How wide was it on the bank ?



12. The trunk of the tree extended two feet on the bank on each side ; how long was it ?

VII. DECIMAL FRACTIONS.

LESSON I.

EXPLAIN to the pupils that in decimal fractions the denominator is not written, but is always understood to be 10, 100, 1000, &c., according to the number of places in the numerator. Explain, also, the use of the *separatrix*, employed to separate the whole number from the decimals. Then, after giving the pupils sufficient practice in the class, to make them in some degree familiar with the notation, assign the following examples :

EXAMPLES.

Write upon the slate, in a perpendicular column, the following numbers.

Three and one tenth.

“ and two tenths.

“ and five tenths.

“ and eight tenths.

“ and nine tenths.

“ and one hundredth.

“ and two hundredths.

“ and five hundredths.

“ and eleven hundredths.

“ and fifteen hundredths.

“ and forty-five hundredths.

“ and ninety-nine hundredths.

Six and one thousandth.
 “ and five thousandths.
 Six and fifty-five thousandths.
 “ and ninety-nine thousandths.
 Seven and ninety-four thousandths.
 “ and four thousandths.
 “ and one ten-thousandth.
 “ and two ten-thousandths.
 “ and three hundred and sixteen ten-thousandths.
 “ and four tenths.
 “ and four hundredths.
 “ and four thousandths.
 Ten and four ten-thousandths.
 Nine and forty-four thousandths.

LESSON II.

ADDITION OF DECIMALS.

All that is necessary for the correct addition of decimals is to place them properly under each other, the separatrix in a column; for, as in whole numbers, units must come under units, tens under tens, &c., so in decimals, tenths must come under tenths, hundredths under hundredths, &c. Then add, as in simple numbers, carrying one for every ten. Thus, in adding 3.06, 1.5, and 6.79 together, we write them thus :—

$$\begin{array}{r}
 3.06 \\
 1.5 \\
 6.79 \\
 \hline
 11.35. \text{ Ans.}
 \end{array}$$

EXAMPLES.

1. Add 3.05, 7.4, 2.1976, and 4.005.
2. " 1.1, 2.000, 3.718, and 4.12.
3. " 1.1, 4.06, 5.14, 0.1 and 1.6.
4. " 3.05, 2.46, 7.111, 8.49 and 123.
5. " 4.4, 5.19, 2.1139, 4.1 and 5.16.
6. " 3.4, 1.005, 8.213, and 6.1.
7. " 4.123, 5.001, 4.367, and 213.01.
8. " 5.201, 0.113, 71.1, and 6.50123.
9. " 3145.1, 21.73, 785.001, and 219876.1.
10. " 21.4, 316.01, 259.01, and 67.113.

11. Add three and one tenth, sixteen and four hundredths, five and one thousandth, and fourteen and eighty-nine thousandths

12. Add four and five tenths, one and twenty-five hundredths, and two and twenty-five hundredths.

LESSON III.

SUBTRACTION OF DECIMALS.

The subtraction of decimals is to be performed in the same manner as in simple numbers, placing the numbers as in addition, with the separatrix of the subtrahend under that of the minuend, so as to bring tenths under tenths, hundredths under hundredths, &c. Of course, where there is no figure in the upper line, a cypher is to be understood; thus :

$$\begin{array}{r}
 \text{From } 20.1 \\
 \text{Take } 3.93. \\
 \hline
 \text{remainder } 16.17.
 \end{array}$$

EXAMPLES.

13.	From	16.1	take	14.36.
14.	"	4.191	"	3.921.
15.	"	8.12	"	7.98.
16.	"	1115.1	"	113.4.
17.	"	6.7	"	5.6.
18.	"	6.6	"	5.7.
19.	"	18.893	"	9.1134567.
20.	"	2.14	"	1.36.
21.	"	3609.09	"	1.9.
22.	"	113.75	"	5.06.
23.	"	22.49	"	5.5.
24.	"	18.1	"	9.9.
25.	"	7.65	"	5.75.
26.	"	8.49	"	3.42.
27.	"	19.111	"	3.645.

28. From seven and eight tenths, take nine hundredths.

29. From one hundred and two and seven tenths, take one hundred and one and nineteen hundredths.

30. From ninety-one and six tenths, take twenty-one and ninety-eight hundredths.

31. From three hundred and six and two hundred and thirty-four thousandths, take twenty-one and seven tenths.

32. From fourteen and four tenths, take five and thirty-nine hundredths.

LESSON IV.

MULTIPLICATION OF DECIMALS.

The pupils are to be taught that, in multiplying decimals, it is not necessary to place the separatrices under each other. The multiplier is to be placed under the multiplicand, and the multiplication performed as if both the numbers were whole numbers. Then a separatrix must be made in the answer in such a place as shall make the number of decimal figures as great in the product as it is in both the factors ; thus :

$$\begin{array}{r}
 \text{Multiply } 6.01 \\
 \text{by } 3.2 \\
 \hline
 1202 \\
 1803 \\
 \hline
 19.232
 \end{array}$$

As there are two decimal places in the multiplicand, and one in the multiplier, we cut off *three* in the product.

EXAMPLES.

33.	Multiply	7.3	by	18.9.
34.	"	1.01	by	2.3.
35.	"	41.3	by	32.001.
36.	"	6.12	by	5.012.
37.	"	3.18	by	5.119.
38.	"	2.5	by	2.5.
39.	"	17.8	by	9.12.
40.	"	4.1	by	3.06.
41.	"	3.51	by	4.09.
42.	"	2.4001	by	3.01.

43.	Multiply	33.3	by	333.
44.	"	4.05	by	5.1.
45.	"	40.5	by	.51.
46.	"	15.5	by	1.55.
47.	"	155.	by	1.55.
48.	"	1.55	by	1.55.
49.	"	271.05	by	2.71.
50.	"	36.34	by	21.56.
51.	"	12.1	by	3.14.
52.	"	14.189	by	22.21.

LESSON V.

It is very important that the pointing off of the decimal places in the answers in such operations, should not be a mere mechanical work; but the pupil should always have in mind a general idea of the true value of the numbers which he is dealing with, and such an expectation in respect to the value of the answer, as shall enable him to act intelligently in determining what are to be the integers of the answer, and to perceive at once his error, if by any accident he should happen to point off a figure too few or too many. To illustrate this take such an example as 7.1 multiplied by 2.12. Here the multiplicand is a little more than 7, and the multiplier a little more than 2. Of course the answer will be something more than 14. Now the figures of the answer obtained by multiplication being 15052, it will be obvious at once, to any one who carries the nature of the operation in his mind, that the first two figures, viz., 15, must be the whole number, and that the answer must be pointed thus: 15.052; and not 1.5052, nor 150.52. For it would be very probable that a number a little greater than seven multiplied by one a little greater than two, would give 15, while it is not possible that it could give 1. or 150. In the same manner, if we multiply 12.459 by 1.101, the figures of the answer being 13717359, it is plain that the first two, 13, must be the whole number, for we had to multiply a little more than 12 by a little more than 1. *Con-*

ing off the decimals in the manner heretofore explained, will give the same result, but it is very important, both to the formation of the habit of *intelligent* working in all such cases, and also as a security against mistakes, that the pupil should always carry in his mind some general idea of the values which he is operating upon, and of the results he expects to obtain. In the following examples, let the pupil, after performing the multiplication, consider what portion of the figures of the result ought to be integers, and point them off according to his judgment, and then afterwards point off the decimals as before directed, to verify the result.

EXAMPLES.

53.	Multiply	3.1	by	4.3.
54.	"	2.113	by	7.001.
55.	"	2.0003	by	5.11102.
56.	"	2.1	by	10.3.
57.	"	21.	by	10.3.
58.	"	.21	by	1.03.
59.	"	300.3	by	2.1.
60.	"	300.3	by	2.2.
61.	"	300.3	by	2.5.
62.	"	300.3	by	2.8.
63.	"	300.3	by	2.9.
64.	"	4.103	by	6.13.
65.	"	14.113	by	5.109.
66.	"	1.06	by	11.07.
67.	"	1.003	by	1.012.
68.	"	1.019	by	2.306.
69.	"	.45	by	2.111.
70.	"	7.16	by	3.17.
71.	"	4.17	by	4.15.
72.	"	31.31	by	2.11.

LESSON VI.

DIVISION OF DECIMALS.

The pupils should be first taught to carry out the division of whole numbers to decimal places by annexing cyphers to the right of the units column; thus :

$$4) 1341(335.25. \text{Ans.}$$

$$\begin{array}{r} 12 \\ \hline 14 \\ 12 \\ \hline 21 \\ 20 \\ \hline 1.0 \\ .8 \\ \hline .20 \end{array}$$

They should be taught to observe very carefully where they begin to introduce decimal places, and place a separatrix in the answer at that point, so as to make all the figures decimals in the quotient, which arise from the divisions of decimal portions of the dividend. For this purpose, they must carefully mark the separatrix among the figures brought down and subtracted in the course of the process as above. The answers may be carried to five decimal places.

EXAMPLES.

73. Divide 443 by 3.
 74. " 3642 by 13.
 75. " 246 by 9.

76.	Divide	2819	by	10.
77.	"	10139	by	101.
78.	"	3165	by	23.
79.	"	489	by	118.
80.	"	4137	by	21.
81.	"	450	by	25.
82.	"	650	by	78.
83.	"	395	by	15.
84.	"	4641	by	4642.
85.	"	246	by	19.
86.	"	2194	by	37.
87.	"	357	by	18.
88.	"	1146	by	21.
89.	"	567	by	31.

LESSON VII.

In case there is already a decimal in the dividend, the process the same as above, the decimal place in the quotient commencing where the first decimal place in the division is used in obtaining quotient figure; thus :

$$12)416.3(34.69, \text{ \&c.}$$

36

56

48

8.3

7.2

1.10

1.08

EXAMPLES.

90.	Divide	4.1	by	2.
91.	"	35.7	by	16.
92.	"	224.5	by	15.
93.	"	3.002	by	5.
94.	"	3.002	by	2.
95.	"	3.002	by	10.
96.	"	45.67	by	44.
97.	"	45.67	by	45.
98.	"	45.67	by	46.
99.	"	327.44	by	23.
100.	"	21.21	by	22.
101.	"	3.1	by	30.
102.	"	7.67	by	15.
103.	"	3.19	by	9.
104.	"	37.241	by	8.
105.	"	21.34	by	7.
106.	"	3.19	by	14.
107.	"	4.21	by	9.
108.	"	3.109	by	3.
109.	"	7.421	by	5.
110.	"	7.413	by	2.
111.	"	11.254	by	16.

LESSON VIII.

In the examples given in the last lessons there have never been more decimal places in the divisor than in the dividend. If, in any case, there are more, the number of decimal places in the dividend must be made equal to that of the divisor, by adding cyphers. The figures of the quotient will then be whole numbers, until the

dividend, increased as above, is exhausted. Other cyphers may afterwards be appended, and the quotient figures arising from them will be decimals; thus :

Divide 116.3 by 4.012.

Here the dividend, having fewer cyphers than the divisor, we add two cyphers, to make the number equal, and then divide, the quotient figures arising being whole numbers, until all the cyphers thus added are exhausted.

$$\begin{array}{r}
 4.012 \overline{) 116.300} \ 28. \\
 \underline{80.24} \\
 36.060 \\
 \underline{32.096} \\
 3.964
 \end{array}$$

We can afterwards continue the division by appending more cyphers, but all the figures of the quotient thus arising will be decimals; thus :

$$\begin{array}{r}
 4.012 \overline{) 116.300} \ 28.8, \text{ \&c.} \\
 \underline{80.24} \\
 36.060 \\
 \underline{32.096} \\
 3.9640 \\
 \underline{3.2096} \\
 .75440
 \end{array}$$

Let the pupil observe the general principle that the whole number in the quotient ends, and the decimals begin, at the point in the

division immediately after where the number of the decimal places of the dividend just equal those of the divisor.

The division in the following examples may be carried to four decimal places.

EXAMPLES.

112.	Divide	34.12	by	3.75.
113.	"	2.5	by	1.001.
114.	"	163.1	by	15.13.
115.	"	98.05	by	1.16.
116.	"	21.1	by	.211.
117.	"	23.6	by	4.16.
118.	"	3982.5	by	1.05.
119.	"	32.19	by	3.012.
120.	"	564.1	by	5.02.
121.	"	36.3	by	.363.
122.	"	115.	by	1.5.
123.	"	99.2	by	1.234.
124.	"	3.	by	.00003.
125.	"	4.	by	.0004.
126.	"	1.	by	.01.
127.	"	21.	by	.21.
128.	"	6.5	by	.00065.
129.	"	35.2	by	.03.
130.	"	225.	by	.109.
131.	"	180.5888	by	161.24.

LESSON IX.

It will be of great service to the pupil to fix in his mind the simple principle which covers all the above cases of division, and, in

fact, all possible cases of division of decimals, that the decimal point in the divisor always comes immediately after that figure of the quotient which was obtained by a division when the decimal places in the dividend and divisor were equal. This principle once fully possessed by the mind of the pupil, will be easily retained and applied, and it covers the whole ground.

Suppose, for example, we have to divide 43.123 by 2.1, we divide thus, until we get to the point where the number of decimal places are equal in the dividend and divisor.

$$\begin{array}{r} 2.1 \overline{) 43.123(20.} \\ \underline{42.} \\ 1.1 \end{array}$$

At this point we place the separatrix in the quotient: and, if we continue the operation, all the figures arising afterwards are decimals. In the same manner in the following case:

$$\begin{array}{r} 2.1 \overline{) .3462(0.1} \\ \underline{0} \\ 34 \end{array}$$

There being one decimal place in the divisor, we take one in the dividend, and the quotient being 0. we place the separatrix immediately after it, and then proceed with the division, all the succeeding figures of the quotient being decimals.

And so in a case like the following, where we have to add cyphers to the dividend in the first instance, to make the number of decimal places in it equal to those in the divisor.

Divide 1. by .01.

$$\begin{array}{r} .01 \overline{) 1.00(100.} \\ \underline{1.} \\ .00 \end{array}$$

We put the separatrix in the quotient immediately after having brought down as many decimal places as there are such places in the divisor. This gives 100 for the answer, which is obviously right, since dividing 1 by .01 is dividing 1 by $\frac{1}{100}$, and $\frac{1}{\frac{1}{100}}$ is contained obviously 100 times in 1.

Let the pupils perform the following examples by means of the above principle. That is, putting the separatrix in the quotient, to mark the commencement of the decimal figures, immediately after the division has proceeded to as many decimal places in the dividend as shall be just equal to the number of decimal places in the divisor. The quotients should be carried to four decimal places.

EXAMPLES.

- | | |
|------|--|
| 132. | How many times are 2.1 contained in 16.3 ? |
| 133. | “ “ 14. “ 15.36 ? |
| 134. | “ “ 1.9999 “ 2. ? |
| 135. | “ “ 1.5 “ 3. ? |
| 136. | “ “ 478.34 “ 3694.139 ? |
| 137. | “ “ 21.1 “ 67.52 ? |
| 138. | “ “ 4.5 “ 3.4521 ? |
| 139. | “ “ 1.1 “ 11. ? |
| 140. | “ “ 1.1 “ 111. ? |
| 141. | “ “ 1.1 “ .11 ? |
| 142. | “ “ 3.4 “ 3.459 ? |
| 143. | Divide 100.6 by .52. |
| 144. | “ 45.1934 by 2. |
| 145. | “ 36.9784 by 3. |
| 146. | “ 45.78 by 31.2 |
| 147. | “ 16.45 by 3.97. |
| 148. | “ 21.39411 by 3. |
| 149. | “ 21.39411 by .3 |
| 150. | “ 21.39411 by .03 |

151. Divide 40. by 3.

152. " 50. by 4.

LESSON X.

TO REDUCE A VULGAR FRACTION TO A DECIMAL.

To reduce a vulgar fraction to a decimal, we have only to divide the numerator by the denominator, adding as many decimal places as may be necessary. Thus, if we are required to reduce $\frac{3}{4}$ to a decimal form, we write—

$$\begin{array}{r} 4 \overline{) 3.0(.75.} \text{ Ans.} \\ \underline{28} \\ 20 \\ \underline{20} \end{array}$$

Thus the answer is .75, that is, seventy-five hundredths. And it is plain that seventy-five hundredths is equal to three quarters. In the following examples let the work be carried to four decimal places:

EXAMPLES.

153. Reduce $\frac{1}{4}$ to the decimal form.

- | | |
|----------------------|----------------------|
| 154. $\frac{1}{3}$. | 162. $\frac{4}{5}$. |
| 155. $\frac{2}{3}$. | 163. $\frac{1}{4}$. |
| 156. $\frac{1}{4}$. | 164. $\frac{3}{8}$. |
| 157. $\frac{3}{4}$. | 165. $\frac{2}{5}$. |
| 158. $\frac{3}{4}$. | 166. $\frac{1}{4}$. |
| 159. $\frac{1}{5}$. | 167. $\frac{5}{8}$. |
| 160. $\frac{3}{4}$. | 168. $\frac{1}{7}$. |
| 1. $\frac{3}{7}$. | 169. $\frac{2}{7}$. |

170. $\frac{3}{4}$.	172. $\frac{3}{4}$.
171. $\frac{3}{4}$.	173. $\frac{3}{4}$.

LESSON XI.

Explain to the pupils the reason for the above process, as follows : A decimal fraction is one which has for its denominator 1 with cyphers annexed. If, therefore, we wish to change a vulgar fraction so as to make it a decimal, what is necessary to accomplish that object is to make the denominator 1 without altering the value. We accordingly divide the denominator by itself, to make it 1, then in order to preserve the value of the fraction unchanged, we must divide the numerator by the same number as that with which we divided the denominator, viz., the denominator itself. We add decimal cyphers to the numerator in carrying out the division, but then as many cyphers are added to the denominator, which is understood, and thus the value of the fraction is not disturbed. The operation in full would be thus :

$$\frac{3}{4} = \frac{3 \overline{) 3.00}}{1 \overline{) 1.00}} = \frac{75}{100} = .75.$$

When this is fully understood by the class, assign the following examples to be performed in full, as in the illustration above. The pupils should, in each case, add as many cyphers to both numerator and denominator, as will be necessary to make the numerator divisible by the original denominator.

EXAMPLES.

Reduce the following fractions to a decimal form, giving the operation in full :

174. $\frac{1}{4}$.	177. $\frac{1}{4}$.
175. $\frac{3}{4}$.	178. $\frac{3}{4}$.
176. $\frac{3}{4}$.	179. $\frac{1}{4}$.

180. $\frac{1}{2}$.

182. $\frac{1}{2}$.

181. $\frac{1}{2}$.

183. $\frac{1}{2}$.

LESSON XII.

REPEATING DECIMALS

In many cases, in dividing the numerator of a fraction by the denominator, we should never come to an end, but the figures of the quotient go on repeating indefinitely. Sometimes one figure repeats, and sometimes more.

Thus, in reducing $\frac{1}{3}$ to a decimal fraction, we get .33333, &c. In reducing the fraction $\frac{1}{3}$, we get for the decimal, .121212, &c., indefinitely. Whenever, in dividing, we come to any remainder the same with any one we have had before, we may know that the figures will, from that time, commence to repeat, so as to give us again the same series that we had, beginning where that remainder occurred before. When such repeating decimals occur, it is customary to denote them by a dot over the repeating figure, or over the first and last, if there are more than one. Thus, 1. $\dot{3}$ means 1.333, &c., without end; and .201 $\dot{2}$ means .20121212, &c., without end.

When the decimal places are not carried out to the end, and do not begin to repeat, it is common to put the sign + after the last figure obtained, to denote that something more remains.

The pupil is to reduce the following fractions to decimals, marking them as repeating decimals when they become so, otherwise adding the sign +, unless the division is carried out to the end. The pupils may carry out the work to as many decimal places as they individually please.

EXAMPLES.

184. $\frac{1}{2}$.

186. $\frac{1}{2}$.

185. $\frac{1}{2}$.

187. $\frac{1}{2}$.

188. $\frac{1}{3}$.	196. $\frac{2}{13}$.
189. $\frac{2}{3}$.	197. $\frac{7}{6}$.
190. $\frac{4}{5}$.	198. $\frac{3}{7}$.
191. $\frac{7}{23}$.	199. $\frac{8}{13}$.
192. $\frac{1}{6}$.	200. $\frac{2}{3}$.
193. $\frac{1}{3}$.	201. $\frac{1}{6}$.
194. $\frac{4}{7}$.	202. $\frac{7}{25}$.
195. $\frac{1}{11}$.	203. $\frac{1}{5}$.

LESSON XIII.

TO REDUCE A DECIMAL TO A VULGAR FRACTION.

As a decimal fraction is only a vulgar fraction with the denominator understood, all that is necessary to give it the form of a vulgar fraction is to express the denominator. Thus, .25 is $\frac{25}{100}$. This may be reduced to its lowest terms, thus :

$$\frac{\overset{5}{25}}{\overset{5}{100}} = \frac{\overset{5}{5}}{\overset{5}{20}} = \frac{1}{4}. \quad \text{Thus .25 is ascertained to be } \frac{1}{4}.$$

EXAMPLES.

Reduce the following decimals to vulgar fractions and to their lowest terms:

204. .25.	212. .125.
205. .025.	213. .0125.
206. .0025.	214. .75.
207. .525.	215. .075.
208. .36.	216. .225.
209. .1884.	217. .8.
210. .0001.	218. .88.
211. .782.	219. .9.

6*

220. .999.

222. .255.

221. .805.

223. .785.

LESSON XIV.

TO REDUCE A REPEATING DECIMAL TO A VULGAR
FRACTION.

If all the figures repeat, we write for a denominator as many nines as there are decimal places in the repetend. Thus, $.3$ is equal to $\frac{3}{10}$ and $.6\dot{3}$ is equal to $\frac{63}{100} = \frac{7}{11}$.

The reason for the above process is this. When we write a repeating decimal we do not express the numerator in full. Thus, in the repeating decimal $.3$, the figure 3 is not the full numerator, but only the first figure of it—the real numerator being 333, &c., without end. Now if, in writing a fraction, we suppress any part of either of the terms, we must diminish the other in precisely the same proportion, in order to preserve the value unchanged; and it is found by certain mathematical calculations, that if we write only the first 3 of the numerator of the decimal $.333$, &c., omitting the rest, and then write 9 instead of 10, for the denominator, we shall diminish both terms in exactly the same proportion, and of course preserve the value of the fraction unchanged. The same reasoning will apply to repeating decimals of any greater number of places. Thus $.301$ is found to be exactly $\frac{301}{999}$. And so in other cases, the denominator of the fraction being always as many nines as there are figures in the repetend.

Of course, if the vulgar fraction obtained by the above process be reduced again to a decimal form, it will give the original repeating decimal. Thus, $.3$ is $\frac{3}{10} = .3$. And $\frac{1}{3}$ reduced to a decimal becomes $.333$, &c., which is to be written $.3$. In the same manner $.27$ is $\frac{27}{100} = \frac{3}{11}$; which brought back to the decimal form by dividing the numerator by the denominator, gives $.2727$, &c. = $.2\dot{7}$, as at first.

EXAMPLES.

224. Reduce $\dot{.30\dot{3}}$ to a vulgar fraction and to its lowest terms, and then back to its original form.

225. $\dot{.40\dot{1}}$.	236. $\dot{.20\dot{1}}$.
226. $\dot{.24\dot{2}}$.	237. $\dot{.19\dot{8}}$.
227. $\dot{.7\dot{2}}$.	238. $\dot{.027}$.
228. $\dot{.2\dot{7}}$.	239. $\dot{.23\dot{1}}$.
229. $\dot{.10\dot{2}}$.	240. $\dot{.13\dot{2}}$.
230. $\dot{.10\dot{3}}$.	241. $\dot{.6\dot{3}}$.
231. $\dot{.11211\dot{3}}$.	242. $\dot{.06\dot{3}}$.
232. $\dot{.8\dot{1}}$.	243. $\dot{.006\dot{3}}$.
233. $\dot{.1\dot{9}}$.	244. $\dot{.301\dot{0}}$.
234. $\dot{.98\dot{1}}$.	245. $\dot{.33\dot{1}}$.
235. $\dot{.71\dot{1}}$.	246. $\dot{.3\dot{6}}$.

LESSON XV.

TO REDUCE A COMPOUND REPETEND TO A VULGAR FRACTION.

A *compound* repetend is one in which the repeating does not commence with the first figure of the decimal, there being one or more figures which do not repeat. For example, $.201\dot{2}$ is a compound repetend. In reducing it to a vulgar fraction we operate upon each portion separately. The part which does not repeat is evidently $\frac{20}{100}$. The part which does repeat would be $\frac{1}{9}$ if it had been a simple repetend; that is, if it had immediately succeeded the decimal point. But by being two places to the right, it is but one-hundredth part of $\frac{1}{9}$; that is, $\frac{1}{900}$. If we would, we may always add to the denominator of $\frac{1}{9}$ 100 .

mal, as many cyphers as the first figure which repeats is removed to the right of the separatrix. Thus, $11\dot{2}$ is $\frac{11}{100}$ and $\dot{4}31\dot{9}$ is $\frac{4}{100}$ and $\frac{319}{1000}$, and so in all other cases.

When we have obtained the values of the two portions of the decimal as above described, we add them together, first expanding the one whose denominator is smallest. The result is the vulgar fraction corresponding in value to the given compound repetend. If, now, this vulgar fraction is reduced to a decimal, it will always exactly reproduce the original compound repetend.

Work upon the Slate.

(247.) $.41\dot{3}\dot{6}$.

$$\frac{41}{100} + \frac{36}{1000}.$$

$$\frac{41}{100} + \frac{114}{1000}.$$

$$\frac{414}{1000} + \frac{114}{1000} = \frac{528}{1000} = \frac{264}{500}.$$

Thus $.41\dot{3}\dot{6}$ is equal to the vulgar fraction $\frac{264}{500}$.

Then, to reduce it back again to its original form, we say

$$\begin{array}{r} 220 \overline{) 91.0} (.41\dot{3}\dot{6} \\ 88.0 \end{array}$$

$$3.00$$

$$2.20$$

$$.800$$

$$.660$$

$$1400$$

$$1320$$

$$800$$

The remainder 800, being the same as one occurring before, the decimal will repeat from this point, and we want the quotient figures accordingly from the place where this last remainder first occurs. In the following examples, the compound repetends are to be reduced to vulgar fractions, and then back again to the original form :

EXAMPLES.

247.	$.41\dot{3}6.$	256.	$.10\dot{3}.$
248.	$.111\dot{2}1.$	257.	$.1\dot{0}3.$
249.	$.23\dot{2}4.$	258.	$.20\dot{1}.$
250.	$.30\dot{0}1.$	259.	$.1\dot{0}2.$
251.	$.10\dot{2}0.$	260.	$.301\dot{2}.$
252.	$.10\dot{2}01.$	261.	$.301\dot{2}.$
253.	$.1\dot{2}.$	262.	$.301\dot{2}.$
254.	$.1\dot{3}.$	263.	$.301\dot{2}.$
255.	$.1\dot{0}3.$	264.	$.105\dot{3}.$

QUESTIONS.

265. Two young men bought a piece of wild land in partnership. They agreed to subdue and cultivate it, and after five years sell it for a farm. They were to divide the money which they obtained for the farm in proportion to the amount they severally paid for the land, and the proceeds of the crops in proportion to the time they each labored. A, one of the young men, paid $\frac{2}{3}$ of the price of the land, and B $\frac{1}{3}$. Afterwards A bought one-quarter of B's share. A labored on the farm 8 hours a-day,

end of 5 years the farm sold for \$1,500. The land had cost \$800. The proceeds of the crops during the whole five years amounted to \$810. On a final settlement, what did each one receive over and above his payment for the land ?

266. A merchant died, and the administrator of his estate found it took all of his property to pay his debts, except a share of $\frac{2}{3}$ which he owned in a ship worth \$60,000. This was to be divided among his heirs. He had had four children, among whom it would have been divided equally ; but one of them, George, had died, and consequently his share was to go to his children, of whom there were three. What proportion of the whole ship would each one of these grandchildren own, and what would be the value of his share ?

ANSWERS.

GENERAL NATURE OF FRACTIONS.

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
35	$\frac{1}{5}$	61	$\frac{14}{8} = \frac{7}{4}$	87	$\frac{9}{7}$	113	$\frac{3}{10}$
36	$\frac{2}{15}$	62	$\frac{6}{4} = \frac{3}{2}$	88	$\frac{9}{7}$	114	$\frac{1}{10}$
37	$\frac{4}{15}$	63	$\frac{2}{10} = \frac{1}{5}$	89	$\frac{5}{3}$	115	$\frac{3}{15}$
38	$\frac{6}{15}$	64	$\frac{4}{6} = \frac{2}{3}$	90	$\frac{15}{6} = \frac{5}{2}$	116	$\frac{6}{15}$
39	$\frac{12}{15}$	65	$\frac{10}{20} = \frac{1}{2}$	91	$\frac{20}{6}$	117	$\frac{4}{3}$
40	$\frac{14}{20}$	66	$\frac{6}{10} = \frac{3}{5}$	92	$\frac{10}{6} = \frac{5}{3}$	118	$\frac{2}{3}$
41	$\frac{10}{15}$	67	$\frac{14}{20} = \frac{7}{10}$	93	$\frac{27}{8}$	119	$\frac{9}{10}$
42	$\frac{6}{10}$	68	$\frac{20}{100} = \frac{1}{5}$	94	$\frac{24}{8} = \frac{3}{1}$	120	$\frac{1}{40}$
43	$\frac{6}{8}$	69	$\frac{10}{100} = \frac{1}{10}$	95	$\frac{4}{8} = \frac{1}{2}$	121	$\frac{2}{40}$
44	$\frac{8}{10}$	70	$\frac{2}{25} = \frac{1}{12.5}$	96	$\frac{35}{8}$	122	$\frac{2}{15}$
45	$\frac{4}{20}$	71	$\frac{6}{8} = \frac{3}{4}$	97	$\frac{28}{8} = \frac{7}{2}$	123	$\frac{15}{15}$
46	$\frac{2}{10}$	72	$\frac{3}{6} = \frac{1}{2}$	98	$\frac{21}{8}$	124	$\frac{2}{9}$
47	$\frac{3}{4}$	73	$\frac{6}{10} = \frac{3}{5}$	99	$\frac{14}{3} = \frac{7}{1.5}$	125	$\frac{1}{9}$
48	$\frac{1}{3}$	74	$\frac{20}{10} = \frac{2}{1}$	100	$\frac{10}{120} = \frac{1}{12}$	126	$\frac{2}{15}$
49	$\frac{1}{3}$	75	$\frac{6}{9} = \frac{2}{3}$	101	$\frac{10}{100} = \frac{1}{10}$	127	$\frac{3}{15} = \frac{1}{5}$
50	$\frac{3}{5}$	76	$\frac{5}{15} = \frac{1}{3}$	102	$\frac{10}{100} = \frac{1}{10}$	128	$\frac{2}{10} = \frac{1}{5}$
51	$\frac{4}{4}$	77	$\frac{10}{15} = \frac{2}{3}$	103	$\frac{10}{15} = \frac{2}{3}$	129	$\frac{4}{14} = \frac{2}{7}$
52	$\frac{1}{4}$	78	$\frac{15}{15} = \frac{1}{1}$	104	$\frac{4}{20} = \frac{1}{5}$	130	$\frac{4}{16} = \frac{1}{4}$
53	$\frac{1}{5}$	79	$\frac{16}{10} = \frac{8}{5}$	105	$\frac{16}{18}$	131	$\frac{9}{30} = \frac{3}{10}$
54	$\frac{6}{10}$	80	$\frac{40}{10} = \frac{4}{1}$	106	$\frac{3}{10}$	132	$\frac{11}{165} = \frac{1}{15}$
55	$\frac{3}{5}$	81	$\frac{3}{15} = \frac{1}{5}$	107	$\frac{2}{10}$	133	$\frac{6}{63} = \frac{2}{21}$
56	$\frac{4}{3}$	82	$\frac{0}{12} = \frac{0}{4}$	108	$\frac{1}{9}$	134	$\frac{21}{133} = \frac{3}{19}$
57	$\frac{5}{8}$	83	$\frac{18}{12} = \frac{3}{2}$	109	$\frac{2}{9}$	135	$\frac{14}{133} = \frac{2}{19}$
58	$\frac{8}{8}$	84	$\frac{6}{12} = \frac{1}{2}$	110	$\frac{5}{9}$	136	$\frac{25}{150} = \frac{1}{6}$
59	$\frac{1}{2}$	85	$\frac{12}{6} = \frac{2}{1}$	111	$\frac{3}{10}$	137	$\frac{25}{100} = \frac{1}{4}$
60	$\frac{6}{9} = \frac{2}{3}$	86	$\frac{8}{4} = \frac{2}{1}$	112	$\frac{5}{10}$	138	$\frac{18}{126} = \frac{1}{7}$

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
139	$\frac{1}{7} \div \frac{2}{3} = \frac{3}{14}$	154	$\frac{2}{3} = \frac{1}{\frac{3}{2}}$	168	$\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$	179	$\frac{2}{7} \div \frac{2}{3} = \frac{3}{7}$
140	$\frac{2}{3} = \frac{1}{\frac{3}{2}}$	155	$\frac{2}{1} \div \frac{2}{3} = \frac{3}{1}$	169	$\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$	180	$\frac{4}{7} \div \frac{2}{3} = \frac{6}{7}$
141	$\frac{1}{6} \div \frac{2}{3} = \frac{1}{4}$	156	$\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$	170	$\frac{7}{7} = \frac{1}{1}$	181	$\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$
142	$\frac{1}{4} \div \frac{2}{3} = \frac{3}{8}$	157	$\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$	171	$\frac{1}{8}$	"	$\frac{1}{12}$
143	$\frac{3}{5} \div \frac{2}{3} = \frac{9}{10}$	158	$\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$	172	$\frac{2}{3}$	182	$\frac{1}{12}$
144	$\frac{2}{7} \div \frac{2}{3} = \frac{3}{7}$	159	$\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$	173	$\frac{1}{3}$	183	$\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$
145	$\frac{1}{7} \div \frac{2}{3} = \frac{3}{14}$	160	$\frac{2}{3} \div \frac{2}{3} = \frac{1}{1}$	174	$\frac{1}{3}$	184	$\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$
146	$\frac{2}{3} \div \frac{2}{3} = \frac{1}{1}$	161	$\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$	175	$\frac{1}{3}$	185	$\frac{2}{3} \div \frac{2}{3} = \frac{1}{1}$
147	$\frac{1}{6}$	162	$\frac{1}{7} \div \frac{2}{3} = \frac{3}{14}$	"	$\frac{1}{7}$	186	$\frac{2}{3} \div \frac{2}{3} = \frac{1}{1}$
148	$\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$	163	$\frac{1}{7} \div \frac{2}{3} = \frac{3}{14}$	"	$\frac{1}{7}$	"	$\frac{1}{7}$
149	$\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$	164	$\frac{1}{7} \div \frac{2}{3} = \frac{3}{14}$	"	$\frac{1}{7}$	187	$\frac{1}{7}$
150	$\frac{3}{4}$	165	$\frac{2}{3} \div \frac{2}{3} = \frac{1}{1}$	176	$\frac{4}{3}$	188	$\frac{1}{3}$
151	$\frac{4}{5} \div \frac{2}{3} = \frac{6}{5}$	166	$\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$	177	$\frac{3}{4}$	189	$\frac{2}{7}$
152	$\frac{4}{5}$	167	$\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$	178	$\frac{1}{4}$	190	$\frac{1}{3}$
153	$\frac{1}{3}$						

CHANGES OF FORM IN FRACTIONAL EXPRESSIONS.

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
1	$\frac{2}{3}$	16	$\frac{1}{5}$	31	$\frac{8}{10}$	46	$\frac{2}{3}$
2	$\frac{3}{4}$	17	$\frac{2}{10}$	32	$\frac{3}{8}$	47	$\frac{1}{2}$
3	$\frac{4}{5}$	18	$\frac{2}{5} \div \frac{3}{5} = \frac{2}{3}$	33	$\frac{3}{4}$	48	$\frac{1}{4}$
4	$\frac{6}{8}$	19	$\frac{4}{8}$	34	$\frac{8}{7}$	49	$\frac{1}{3}$
5	$\frac{1}{2}$	20	$\frac{4}{20}$	35	$\frac{1}{1}$	50	$\frac{1}{3}$
6	$\frac{3}{6}$	21	$\frac{1}{2} \div \frac{7}{5} = \frac{5}{14}$	36	$\frac{1}{1}$	51	$\frac{2}{3}$
7	$\frac{2}{4}$	22	$\frac{1}{10} \div \frac{6}{10} = \frac{1}{6}$	37	$\frac{1}{2}$	52	$\frac{1}{10}$
8	$\frac{1}{2}$	23	$\frac{1}{5} \div \frac{6}{5} = \frac{1}{6}$	38	$\frac{5}{2}$	53	$\frac{2}{3}$
9	$\frac{1}{2}$	24	$\frac{4}{4}$	39	$\frac{1}{1}$	54	$\frac{4}{5}$
10	$\frac{2}{3} \div \frac{2}{3} = \frac{1}{1}$	25	$\frac{2}{3}$	40	$\frac{2}{3}$	55	$\frac{2}{3}$
11	$\frac{1}{2}$	26	$\frac{6}{6}$	41	$\frac{2}{1}$	56	$\frac{1}{3}$
12	$\frac{1}{2}$	27	$\frac{2}{3}$	42	$\frac{6}{6}$	57	$\frac{4}{1}$
		28	$\frac{2}{3}$	43	$\frac{2}{3}$	58	$\frac{1}{2}$
		29	$\frac{2}{3}$	44	$\frac{2}{4}$	59	$\frac{2}{3}$
		30	$\frac{1}{2}$	45	$\frac{1}{2}$	60	$\frac{2}{3}$

CHANGES OF FORM IN FRACTIONAL EXPRESSIONS. 161

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
61	$\frac{9}{5}$	96	$\frac{1}{2}$	131	$\frac{4}{3}$	166	$\frac{4}{3}$	201	$\frac{8}{19}$
62	$\frac{9}{5}$	97	$\frac{9}{10}$	132	$\frac{7}{6}$	167	$\frac{25}{4}$	202	$\frac{1}{25}$
63	$\frac{3}{4}$	98	$\frac{3}{40}$	133	$\frac{9}{5}$	168	$\frac{83}{2000}$	203	$\frac{78}{139}$
64	$\frac{1}{3}$	99	$\frac{9}{5}$	134	$\frac{1}{3}$	169	$\frac{19}{10}$	204	$\frac{3}{8}$
65	$\frac{2}{15}$	100	$\frac{1}{2}$	135	$\frac{7}{11}$	170	$\frac{8}{9}$	205	$\frac{5}{29}$
66	$\frac{1}{15}$	101	$\frac{5}{6}$	136	$\frac{1}{17}$	171	$\frac{1}{24}$	206	$\frac{1}{11}$
67	$\frac{1}{30}$	102	$\frac{2}{3}$	137	4	172	$\frac{1}{10}$	207	$\frac{9}{15}$
68	$\frac{5}{10}$	103	$\frac{7}{9}$	138	8	173	$\frac{4}{25}$	208	$\frac{1}{2}$
69	$\frac{1}{2}$	104	$\frac{7}{8}$	139	10	174	$\frac{1}{10}$	209	$\frac{1}{2}$
70	$\frac{3}{19}$	105	$\frac{4}{5}$	140	40	175	$\frac{1}{5}$	210	$\frac{2}{3}$
71	$\frac{4}{13}$	106	$\frac{3}{5}$	141	24	176	$\frac{2}{43}$	211	$\frac{63}{100}$
72	$\frac{1}{3}$	107	$\frac{1}{13}$	142	120	177	$\frac{73}{500}$	212	$\frac{2}{29}$
73	$\frac{1}{2}$	108	$\frac{6}{13}$	143	2	178	$\frac{365}{251}$	213	$\frac{42}{309}$
74	$\frac{1}{2}$	109	$\frac{1}{33}$	144	2	179	$\frac{9}{109}$	214	$\frac{56}{435}$
75	$\frac{1}{2}$	110	$\frac{3}{9}$	145	9	180	$\frac{13}{109}$	215	$\frac{364}{459}$
76	$\frac{4}{5}$	111	$\frac{9}{20}$	146	9	181	$\frac{23}{45}$	216	$\frac{1}{2}$
77	$\frac{3}{4}$	112	$\frac{9}{8}$	147	300	182	$\frac{79}{195}$	217	$\frac{369}{0}$
78	$\frac{1}{7}$	113	$\frac{1}{3}$	148	4	183	$\frac{1}{2}$	218	$\frac{1404}{3}$
79	$\frac{5}{6}$	114	$\frac{3}{10}$	149	20	184	$\frac{51}{200}$	219	$\frac{815}{15}$
80	$\frac{5}{6}$	115	$\frac{1}{3}$	150	66	185	$\frac{14}{29}$	220	$\frac{449}{10}$
81	$\frac{1}{2}$	116	$\frac{2}{3}$	151	100	186	$\frac{75}{180}$	221	$\frac{3520}{48}$
82	$\frac{1}{5}$	117	$\frac{1}{4}$	152	36	187	$\frac{139}{220}$	222	$\frac{7684}{175}$
83	$\frac{1}{5}$	118	$\frac{22}{49}$	153	32	188	$\frac{19}{8}$	223	$\frac{475}{4}$
84	$\frac{1}{9}$	119	$\frac{3}{4}$	154	8	189	$\frac{3}{20}$	224	$\frac{8488}{4}$
85	$\frac{1}{6}$	120	$\frac{15}{10}$	155	14	190	$\frac{1}{30}$	225	$\frac{14489792}{6752}$
86	$\frac{7}{8}$	121	$\frac{9}{10}$	156	$\frac{40}{73}$	191	$\frac{3}{11}$	226	$\frac{8046}{175}$
87	$\frac{8}{9}$	122	$\frac{5}{9}$	157	$\frac{34}{67}$	192	$\frac{3}{5}$	227	$\frac{222}{2}$
88	$\frac{14}{15}$	123	$\frac{3}{43}$	158	$\frac{2}{3}$	193	$\frac{3}{4}$	228	$\frac{7000}{10}$
89	$\frac{7}{8}$	124	$\frac{9}{11}$	159	$\frac{7}{9}$	194	$\frac{1}{8}$	229	$\frac{2550}{16}$
90	$\frac{1}{9}$	125	$\frac{1}{3}$	160	$\frac{56}{9}$	195	$\frac{81}{11}$	230	$\frac{12000}{16}$
91	$\frac{2}{9}$	126	$\frac{7}{14}$	161	$\frac{1}{10}$	196	$\frac{13}{25}$	231	$\frac{62}{2}$
92	$\frac{1}{3}$	127	$\frac{16}{35}$	162	$\frac{1}{20}$	197	$\frac{37}{150}$	232	$\frac{9630}{175}$
93	$\frac{1}{9}$	128	$\frac{3}{3}$	163	$\frac{3}{9}$	198	$\frac{3}{3}$	233	$\frac{80}{1}$
94	$\frac{1}{2}$	129	$\frac{7}{8}$	164	$\frac{15}{40}$	199	$\frac{53}{6}$	234	$\frac{41}{7}$
95	$\frac{14}{15}$	130	$\frac{3}{4}$	165	$\frac{39}{8}$	200	$\frac{31}{6}$	235	$\frac{85}{3}$

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
236	$1\frac{5}{2}7$	269	$17\frac{1}{2}$	302	$\frac{252}{1140}$	MIS. EX.	
237	$3\frac{1}{9}$	270	3	303	$\frac{7062}{336}$	1	$\frac{24}{30}$
238	$3\frac{21}{100}$	271	$1\frac{1}{2}$	304	$\frac{357}{144}$	2	$\frac{2}{3}$
239	$3\frac{9}{16}$	272	$1\frac{1}{3}$	305	$\frac{15}{144}$	3	$\frac{1}{3}$
240	$1\frac{8}{10}$	273	$60\frac{2}{3}$	306	$\frac{12}{12}$	4	$\frac{3}{10}$
241	$3\frac{5}{8}$	274	3360	307	$\frac{1260}{105}$	5	$\frac{30}{40}$
242	$1\frac{1}{6}$	275	$\frac{33}{135}$	308	$\frac{18}{64}$	6	$\frac{5}{40}$
243	$\frac{8}{5}$	276	$\frac{2}{140}$	309	$\frac{133}{133}$	7	900
244	$1\frac{4}{10}$	277	$\frac{19}{405}$	310	$\frac{175725}{240}$	8	$\frac{30}{900}$
245	$\frac{0}{5}$	278	$\frac{1}{2300}$	311	$\frac{7}{225}$	9	$\frac{225}{9000}$
246	$1\frac{6}{10}7$	279	$\frac{19}{300}$	312	$\frac{17}{800}$	10	$\frac{225}{9000}$
247	$2\frac{8}{10}4$	280	$\frac{1}{18}$	313	$\frac{2}{30}$	11	$\frac{1}{4}$
248	$3\frac{0}{10}5$	281	$\frac{1}{45}$	314	$\frac{54}{33}$	12	$\frac{1}{3}$
249	$3\frac{0}{10}6$	282	$\frac{1}{320}$	315	$\frac{4}{9}$	13	$\frac{3}{4}$
250	$4\frac{5}{17}3$	283	$\frac{1}{13000}$	316	$\frac{27}{80}$	14	$\frac{1}{900}$
251	$1\frac{0}{5}4$	284	$\frac{1}{144}$	317	$\frac{8}{13}$	15	$\frac{5}{900}$
252	$3\frac{7}{13}$	285	$\frac{80}{1975}$	318	$\frac{6}{10}$	16	$\frac{1}{12}$
253	$1\frac{4}{12}5$	286	$\frac{2}{1500}$	319	$\frac{1}{90}$	17	$\frac{1}{6}$
254	3	287	$\frac{2}{253}$	320	$\frac{2}{7}$	18	$\frac{4}{15}$
255	$16\frac{1}{20}3$	288	$\frac{18}{384}$	321	$\frac{1}{7}$	19	$\frac{1}{90}$
256	$2\frac{2}{8}$	289	$\frac{1}{900}$	322	$\frac{3}{10}$	20	$\frac{3}{4}$
257	$8\frac{1}{10}4$	290	$\frac{6}{375}$	323	$\frac{1}{9}$	21	$\frac{2}{30}$
258	$9\frac{7}{19}$	291	$\frac{3}{400}$	324	$\frac{6}{35}$	22	$\frac{1}{12}$
259	3	292	$\frac{1}{2340}$	325	$\frac{20}{37}$	23	$\frac{2}{30}$
260	12	293	$\frac{3}{1845504}$	326	$\frac{5}{39}$	24	$\frac{3}{9}$
261	101	294	$\frac{7}{10}$	327	$\frac{9}{30}$	25	$\frac{43}{180}$
262	11	295	$\frac{10}{160}$	328	$\frac{16}{21}$	26	$\frac{37}{60}$
263	$14\frac{4}{9}$	296	$\frac{315}{304}$	329	$\frac{1}{3}$	27	$\frac{182}{30}$
264	497	297	$\frac{2}{195}$	330	$\frac{1}{6}$	28	$5\frac{1}{18}$
265	30	298	$\frac{56}{190}$	331	$\frac{3}{38}$	29	$\frac{1}{18}$
266	22	299	$\frac{1}{180}$	332	$\frac{1}{17}$	30	$\frac{3}{4}$
267	$5\frac{1}{3}$	300	$\frac{21}{30}$	333	$\frac{7}{25}$		
268	$6\frac{4}{9}1$	301	$\frac{364}{4332}$	334	$\frac{1}{2}$		

ADDITION OF FRACTIONS.

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
1	$\frac{13}{5}$	33	1	65	$1\frac{2}{9}$	97	$\frac{83}{150}$	129	$\frac{26}{35}$
2	$\frac{11}{5}$	34	$1\frac{1}{35}$	66	$\frac{7}{30}$	98	$\frac{51}{60}$	130	$\frac{41}{60}$
3	$\frac{15}{20}$	35	$1\frac{1}{17}$	67	$1\frac{1}{8}$	99	$1\frac{2}{15}$	131	$\frac{4}{5}$
4	$\frac{17}{5}$	36	2	68	$1\frac{1}{4}$	100	$\frac{91}{400}$	132	$\frac{9}{20}$
5	$\frac{8}{9}$	37	$1\frac{1}{7}$	69	1	101	$\frac{10}{50}$	133	$1\frac{1}{10}$
6	$\frac{5}{6}$	38	$\frac{1}{5}$	70	$\frac{1}{2}$	102	$\frac{145}{342}$	134	$\frac{2}{5}$
7	$\frac{15}{20}$	39	$\frac{1}{30}$	71	$1\frac{5}{9}$	103	$\frac{43}{50}$	135	$1\frac{1}{20}$
8	$\frac{11}{6}$	40	$\frac{1}{4}$	72	$1\frac{3}{20}$	104	$\frac{10}{45}$	136	$\frac{53}{144}$
9	$\frac{6}{5}$	41	$\frac{1}{2}$	73	$\frac{24}{25}$	105	$\frac{61}{272}$	137	$\frac{5}{8}$
10	$2\frac{1}{8}$	42	$\frac{2}{25}$	74	$1\frac{7}{20}$	106	$\frac{129}{600}$	138	$\frac{109}{140}$
11	$\frac{2}{9}$	43	$1\frac{7}{10}$	75	$\frac{19}{60}$	107	$1\frac{5}{12}$	139	$\frac{1}{2}$
12	$\frac{47}{30}$	44	$1\frac{1}{10}$	76	$\frac{71}{80}$	108	$1\frac{1}{30}$	140	$1\frac{2}{3}$
13	$\frac{32}{10}$	45	$\frac{15}{10}$	77	$1\frac{9}{20}$	109	$1\frac{7}{30}$	141	$\frac{7}{8}$
14	$\frac{8}{5}$	46	$1\frac{1}{8}$	78	$1\frac{5}{40}$	110	$1\frac{31}{40}$	142	$1\frac{1}{4}$
15	$3\frac{71}{10}$	47	$1\frac{1}{10}$	79	$\frac{3}{10}$	111	$2\frac{7}{5}$	143	$\frac{21}{20}$
16	$\frac{0}{0}$	48	$\frac{7}{8}$	80	$\frac{21}{100}$	112	$\frac{10}{21}$	144	$1\frac{7}{5}$
17	$\frac{1768}{1600}$	49	$\frac{7}{8}$	81	$1\frac{2}{3}$	113	$\frac{125}{42}$	145	$\frac{61}{100}$
18	$\frac{4830}{2500}$	50	$\frac{2}{7}$	82	$\frac{61}{100}$	114	$1\frac{1}{2}$	146	$\frac{631}{640}$
19	$\frac{254}{60}$	51	$\frac{2}{3}$	83	$\frac{7}{20}$	115	$1\frac{26}{45}$	147	$\frac{139}{160}$
20	$\frac{6}{9}$	52	$1\frac{7}{20}$	84	$\frac{1}{2}$	116	1	148	20
21	$\frac{4}{70}$	53	$\frac{91}{100}$	85	$1\frac{3}{8}$	117	$1\frac{1}{105}$	149	144
22	$\frac{1}{5}$	54	$1\frac{1}{8}$	86	$1\frac{2}{15}$	118	$1\frac{1}{4}$	150	210
23	$1\frac{3}{5}$	55	$\frac{50}{30}$	87	$\frac{10}{21}$	119	$\frac{928}{1435}$	151	40
24	$1\frac{12}{9}$	56	$1\frac{3}{30}$	88	$1\frac{7}{15}$	120	$\frac{101}{108}$	152	30
25	3	57	$\frac{7}{10}$	89	$\frac{7}{12}$	121	1	153	72
26	2	58	$\frac{3}{20}$	90	$\frac{11}{20}$	122	$1\frac{9}{12}$	154	360
27	1	59	$2\frac{1}{2}$	91	$\frac{40}{90}$	123	2	155	140
28	$1\frac{7}{100}$	60	$2\frac{3}{4}$	92	$\frac{103}{150}$	124	$1\frac{3}{30}$	156	40
29	$1\frac{2}{5}$	61	$5\frac{2}{9}$	93	$\frac{45}{50}$	125	$2\frac{1}{4}$	157	210
30	$\frac{2}{20}$	62	$\frac{7}{4}$	94	$1\frac{1}{6}$	126	$\frac{2}{3}$	158	120
31	$1\frac{661}{180}$	63	$\frac{3}{10}$	95	$1\frac{16}{45}$	127	$1\frac{11}{30}$	159	$1\frac{1}{7}$
32	1	64	$\frac{9}{18}$	96	$\frac{37}{40}$	128	$\frac{17}{16}$	160	1

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
161	$\frac{4}{7}$	170	60	179	4	MISC. EX.		9	$\frac{1}{15}$
162	$1\frac{2}{3}$	171	80	180	$2\frac{7}{10}$	$1\frac{7}{10}$		10	$119\frac{1}{2}$
163	$\frac{1}{3}$	172	22	181	$5\frac{4}{10}$	$26\frac{1}{10}$		11	$\frac{1}{5}$
164	$1\frac{1}{10}$	173	2	182	$3\frac{9}{10}$	$3\frac{1}{10}$		12	$1\frac{1}{2}$
165	$3\frac{3}{10}$	174	$1\frac{8}{15}$	183	$3\frac{9}{10}$	$4\frac{1}{15}$		13	C
166	$1\frac{9}{10}$	175	$4\frac{1}{10}$	184	$2\frac{14}{10}$	$5\frac{7}{15}$		14	$1\frac{1}{10}$
167	$\frac{2}{1}$	176	$2\frac{8}{10}$	185	$\frac{3}{4}$	$6\frac{2}{15}$		15	A little more.
168	$\frac{5}{15}$	177	$\frac{1}{15}$	186	$1\frac{1}{2}$	$7\frac{1}{2}$ of $\frac{1}{3}$			
169	10	178	$9\frac{1}{15}$	187	$\frac{3}{4}$	$8\frac{2}{3}$ of $\frac{1}{10}$			

SUBTRACTION OF FRACTIONS.

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
1	$\frac{4}{10}$	22	$\frac{11}{15}$	43	$\frac{3}{8}$	64	$2\frac{5}{10}$	85	$3\frac{1}{15}$
2	$\frac{9}{10}$	23	$\frac{9}{15}$	44	0	65	$10\frac{7}{10}$	86	$4\frac{1}{10}$
3	$1\frac{1}{10}$	24	$\frac{1}{10}$	45	0	66	$20\frac{6}{10}$	87	$\frac{2}{10}$
4	$2\frac{9}{10}$	25	$\frac{7}{15}$	46	$3\frac{3}{10}$	67	$44\frac{2}{10}$	88	$2\frac{8}{10}$
5	$\frac{16}{10}$	26	$\frac{13}{10}$	47	$1\frac{1}{10}$	68	$\frac{1}{10}$	89	$7\frac{9}{10}$
6	$\frac{3}{10}$	27	$\frac{6}{10}$	48	0	69	$7\frac{1}{10}$	90	$1\frac{3}{10}$
7	$\frac{2}{10}$	28	$\frac{1}{10}$	49	$\frac{7}{10}$	70	$9\frac{3}{10}$	91	$\frac{3}{10}$
8	$\frac{2}{10}$	29	$\frac{1}{10}$	50	$3\frac{1}{10}$	71	$8\frac{0}{10}$	92	$14\frac{3}{10}$
9	$\frac{9}{10}$	30	$2\frac{2}{10}$	51	$4\frac{2}{10}$	72	0	93	0
10	$\frac{5}{10}$	31	$\frac{1}{10}$	52	$4\frac{1}{10}$	73	$8\frac{7}{10}$	94	0
11	$\frac{1}{15}$	32	$\frac{4}{10}$	53	$7\frac{4}{10}$	74	$5\frac{4}{15}$	95	$3\frac{1}{15}$
12	$\frac{4}{12}$	33	0	54	$\frac{1}{10}$	75	$8\frac{1}{10}$	96	$\frac{4}{10}$
13	$\frac{9}{10}$	34	$\frac{3}{10}$	55	$8\frac{2}{10}$	76	$3\frac{1}{10}$	97	$\frac{3}{10}$
14	$\frac{9}{10}$	35	$\frac{8}{10}$	56	$6\frac{7}{15}$	77	2	98	$\frac{1}{10}$
15	$3\frac{1}{10}$	36	$\frac{5}{10}$	57	$\frac{1}{15}$	78	$4\frac{8}{10}$	99	$\frac{5}{10}$
16	$\frac{1}{4}$	37	$\frac{3}{10}$	58	$\frac{1}{8}$	79	$4\frac{1}{10}$	100	$\frac{1}{10}$
17	$\frac{2}{10}$	38	$\frac{1}{10}$	59	$\frac{1}{10}$	80	$189\frac{1}{10}$	101	$\frac{1}{10}$
18	$\frac{1}{10}$	39	$\frac{2}{10}$	60	$\frac{1}{10}$	81	$\frac{3}{10}$	102	$3\frac{3}{10}$
19	$1\frac{1}{10}$	40	$\frac{1}{10}$	61	$7\frac{1}{10}$	82	$2\frac{1}{10}$	103	$10\frac{4}{10}$
20	$3\frac{2}{10}$	41	$\frac{1}{10}$	62	$3\frac{2}{10}$	83	$1\frac{1}{10}$	104	$9\frac{5}{10}$
21	$\frac{3}{10}$	42	$3\frac{1}{10}$	63	$9\frac{1}{10}$	84	$18\frac{1}{10}$	105	18

MULTIPLICATION OF FRACTIONS.

165

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
106	13	111	14 $\frac{1}{7}$	117	20 $\frac{2}{5}$	MISC. EX.		4	M $\frac{17}{80}$
107	2	112	13 $\frac{3}{5}$	118	2	1	6 $\frac{3}{8}$	5	$\frac{1}{4}$
108	$\frac{2}{3}$	113	24	119	5 $\frac{7}{8}$	2	$\frac{1}{15}$	6	$\frac{5}{10}$
109	$\frac{1}{4}$	114	2 $\frac{1}{4}$	120	40	3	W. $\frac{1}{60}$	7	9 $\frac{1}{4}$
110	20	115	1 $\frac{6}{8}$	121	5	4	C. $\frac{4}{60}$	8	16 $\frac{5}{6}$
		116					S. $\frac{1}{60}$	9	A. $\frac{4}{5}$
								13	78 $\frac{1}{8}$

MULTIPLICATION OF FRACTIONS.

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
1	$\frac{1}{3}$	23	$\frac{1}{4}$	45	$\frac{3}{4}$	67	12 $\frac{3}{10}$	89	22 $\frac{3}{10}$
2	$\frac{6}{15}$	24	$\frac{1}{6}$	46	1 $\frac{9}{10}$	68	123 $\frac{3}{7}$	90	40 $\frac{5}{6}$
3	$\frac{7}{10}$	25	143 $\frac{1}{5}$	47	$\frac{7}{4}$	69	$\frac{6}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{1}{10}$	91	5 $\frac{1}{16}$
4	$\frac{17}{20}$	26	62 $\frac{3}{4}$	48	$\frac{2}{3}$	70	$\frac{7}{15}$	92	$\frac{1}{2}$
5	$\frac{1}{2}$	27	$\frac{4}{7}$	49	49	71	1 $\frac{5}{10}$	93	1 $\frac{1}{8}$
6	$\frac{3}{10}$	28	14 $\frac{7}{12}$	50	20	72	$\frac{3}{10}$ $\frac{5}{10}$ $\frac{2}{10}$	94	20 $\frac{5}{16}$
7	$\frac{1}{20}$	29	3 $\frac{5}{9}$	51	14	73	$\frac{2}{10}$	95	3 $\frac{1}{4}$
8	1	30	$\frac{9}{80}$	52	1 $\frac{1}{5}$	74	60	96	58
9	$\frac{1}{14}$	31	792 $\frac{3}{7}$	53	2 $\frac{5}{8}$	75	70	97	14 $\frac{1}{6}$
10	$\frac{1}{20}$	32	$\frac{1}{2}$ $\frac{7}{5}$ $\frac{1}{3}$	54	16 $\frac{1}{2}$	76	78	98	37 $\frac{1}{24}$
11	$\frac{1}{30}$	33	$\frac{1}{2}$ $\frac{8}{3}$	55	35 $\frac{1}{7}$	77	27	99	138773 $\frac{1}{4}$
12	$\frac{1}{3}$ $\frac{5}{9}$	34	3 $\frac{3}{4}$	56	10 $\frac{1}{2}$	78	61 $\frac{5}{7}$	100	3 $\frac{1}{10}$
13	$\frac{2}{10}$	35	8	57	$\frac{6}{1}$	79	$\frac{1}{20}$	101	2131 $\frac{1}{2}$ $\frac{3}{4}$
14	$\frac{7}{10}$	36	3	58	4 $\frac{1}{2}$	80	$\frac{2}{10}$ $\frac{7}{10}$ $\frac{8}{10}$	102	1024 $\frac{8}{15}$
15	1	37	4	59	2 $\frac{4}{5}$	81	17 $\frac{2}{5}$	103	2771 $\frac{5}{16}$
16	$\frac{1}{10}$	38	672	60	4152	82	1	104	15 $\frac{1}{6}$
17	$\frac{2}{3}$	39	41 $\frac{4}{3}$	61	$\frac{4}{6}$ $\frac{8}{6}$ $\frac{1}{6}$	83	15 $\frac{3}{5}$	105	93 $\frac{1}{6}$
18	$\frac{4}{15}$	40	3	62	$\frac{6}{3}$	84	31 $\frac{7}{5}$	106	199 $\frac{2}{5}$ $\frac{3}{6}$
19	$\frac{1}{10}$	41	3	63	$\frac{6}{3}$	85	2 $\frac{2}{7}$	107	21 $\frac{3}{5}$
20	$\frac{1}{4}$	42	9 $\frac{9}{6}$	64	$\frac{9}{10}$ $\frac{3}{10}$	86	34 $\frac{2}{3}$	108	78 $\frac{2}{3}$
21	2 $\frac{2}{5}$	43	$\frac{7}{8}$	65	$\frac{2}{7}$	87	$\frac{9}{15}$ $\frac{6}{6}$	109	89 $\frac{7}{6}$
22	8 $\frac{4}{7}$	44	9 $\frac{1}{2}$	66	1 $\frac{3}{4}$	88	$\frac{8}{2}$ $\frac{3}{3}$	110	11 $\frac{1}{6}$

DIVISION OF FRACTIONS.

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
1	$\frac{2}{5}$	24	2	47	$\frac{1}{6}$	70	4	93	45
2	1	25	$2\frac{1}{7}$	48	$\frac{1}{4}$	71	12	94	$\frac{1}{6}$
3	$1\frac{1}{2}$	26	$2\frac{2}{7}$	49	$\frac{2}{5}$	72	9	95	$\frac{3}{5}$
4	6	27	$2\frac{3}{7}$	50	$1\frac{5}{6}$	73	6	96	$\frac{6}{21}$
5	3	28	$3\frac{2}{3}$	51	$9\frac{1}{5}$	74	49	97	$\frac{2}{3}$
6	12	29	$63\frac{1}{3}$	52	$\frac{1}{5}$	75	63	98	1
7	$2\frac{0}{21}$	30	$66\frac{9}{17}$	53	$\frac{1}{5}$	76	$9\frac{1}{3}$	99	$\frac{3}{4}$
8	$3\frac{1}{13}$	31	19	54	$\frac{7}{8}$	77	4	100	$\frac{1}{12}$
9	$\frac{5}{3}$	32	$123\frac{3}{5}$	55	$\frac{1}{10}$	78	$\frac{1}{4}$	MISC. EX. 120 221C $\frac{7}{10}$ Y 326 $\frac{1}{14}$ 426C $\frac{1}{20}$ Y 55 $\frac{1}{14}$ 6 Boys, 6 4 yards 7 $\frac{1}{4}$ 8 5333 sold. 1 lb. 9 14 10 $\frac{5}{6}$ 11 $15\frac{3}{4}$ 12 19 $\frac{3}{4}$	
10	$\frac{1}{18}$	33	$1\frac{13}{2}$	56	$\frac{1}{4}$	79	$\frac{2}{9}$		
11	2	34	$\frac{1}{19}$	57	$\frac{4}{25}$	80	$\frac{4}{9}$		
12	$\frac{1}{8}$	35	$\frac{1}{2}$	58	$\frac{2}{1}$	81	$\frac{1}{2}$		
13	$\frac{1}{7}$	36	$\frac{2}{3}$	59	$\frac{7}{4}$	82	$\frac{4}{5}$		
14	$\frac{1}{20}$	37	$\frac{7}{9}$	60	$\frac{1}{4}$	83	$\frac{1}{2}$		
15	$1\frac{1}{5}$	38	2	61	$18\frac{2}{3}$	84	$\frac{2}{3}$		
16	$\frac{3}{5}$	39	$\frac{3}{4}$	62	95	85	$\frac{3}{5}$		
17	$1\frac{2}{3}$	40	1	63	8900	86	$\frac{5}{7}$		
18	$3\frac{1}{3}$	41	$\frac{7}{4}$	64	6	87	$1\frac{1}{2}$		
19	$\frac{1}{12}$	42	$\frac{3}{25}$	65	289	88	1		
20	$\frac{1}{15}$	43	$\frac{4}{5}$	66	16	89	$\frac{1}{3}$		
21	10	44	$3\frac{4}{25}$	67	741	90	$\frac{1}{3}$		
22	2	45	$\frac{1}{5}$	68	25	91	$\frac{7}{54}$		
23	2	46	$\frac{1}{5}$	69	440	92	$\frac{1}{4}$		

DECIMAL FRACTIONS.

No.	Ans.	No.	Ans.	No.	Ans.	No.	Ans.
ADDITION.		6	18.718	12	8.00	17	1.1
1	16.6526	7	226.501	SUBTRACTION.		18	0.9
2	10.938	8	82.91523	13	1.74	19	9.7795433
3	12.00	9	223827.931	14	0.270	20	0.78
4	144.111	10	663.533	15	0.14	21	3607.19
5	20.9639	11	38.230	16	1001.7	22	108.69

No	Ans.	No	Ans.	No	Ans.	No	Ans.
23	16.99	60	660.66	97	1.01488	135	2
24	8.2	61	750.75	98	0.99282	136	7.7228
25	1.90	62	840.84	99	14.23652	137	3.2
26	5.07	63	870.87	100	0.96409	138	0.7671
27	15.466	64	25.15139	101	.103333	139	10
28	7.71	65	72.103317	102	0.51133	140	100.9090
29	1.51	66	11.7342	103	0.35444	141	0.1
30	69.62	67	1.015036	104	4.65512	142	1.0173
31	284.534	68	2.349814	105	3.04857	143	193.4615
32	9.01	69	0.94995	106	0.22785	144	22.5967
MULTIPLICATION.		70	22.6972	107	0.46777	145	12.3261
33	137.97	71	17.3055	108	1.03633	146	1.4673
34	2.323	72	66.0641	109	1.4842	147	0.4143
35	1321.6413	DIVISION.		110	3.7065	148	7.13137
36	30.67344	73	147.66666	111	0.70337	149	71.3137
37	16.27842	74	280.15386	112	9.9866	150	713.137
38	6.25	75	27.33333	113	2.4975	151	13.3333
39	162.336	76	281.9	114	10.7799	152	12.5
40	12.546	77	100.38613	115	84.5253	153	0.5
41	14.3559	78	137.60869	116	100	154	0.3333
42	7.224301	79	4.14406	117	5.6730	155	.6666
43	11088.9	80	197	118	3792.8571	156	25
44	20.655	81	18	119	10.6872	157	.5
45	20.655	82	8.33333	120	112.3705	158	.75
46	24.025	83	26.33333	121	100	159	.2
47	240.25	84	0.99978	122	79.3333	160	.4
48	2.4025	85	12.94736	123	80.3889	161	.6
49	734.5455	86	59.29729	124	100000	162	.8
50	783.4904	87	19.83333	125	10000	163	.1666
51	37.994	88	54.57142	126	100	164	.3333
52	315.13769	89	18.29032	127	100	165	.5
53	13.33	90	2.05	128	10000	166	.6666
54	14.793113	91	2.23125	129	1173.3333	167	.8333
55	10.223573306	92	14.96666	130	2064.2201	168	.1428
56	21.63	93	0.6004	131	1.12	169	.2857
57	216.3	94	1.501	132	7.7619	170	.4285
58	0.2163	95	0.3002	133	1.0971	171	.5714
59	630.63	96	1.03795	134	1.0000	172	.7142

No.	Ans.	No.	Ans.	No.	Ans.
173.8571		197.7		221 $\frac{1}{3} \frac{1}{3}$	245 $\frac{2}{3} \frac{1}{3}$
174.2		198.4285+		222 $\frac{2}{3} \frac{1}{3}$	246 $\frac{1}{3} \frac{1}{3}$
175.4		199.53		223 $\frac{1}{3} \frac{1}{3}$	247 $\frac{2}{3} \frac{1}{3}$
176.6		200.92		224 $\frac{2}{3} \frac{1}{3}$	248 $\frac{2}{3} \frac{1}{3}$
177.25		201.9473		225 $\frac{2}{3} \frac{1}{3}$	249 $\frac{2}{3} \frac{1}{3}$
178.75		202.35		226 $\frac{2}{3} \frac{1}{3}$	250 $\frac{2}{3} \frac{1}{3}$
179.7		203.0526		227 $\frac{2}{3} \frac{1}{3}$	251 $\frac{2}{3} \frac{1}{3}$
180.08		204 $\frac{1}{3}$		228 $\frac{2}{3} \frac{1}{3}$	252 $\frac{2}{3} \frac{1}{3}$
181.15		205 $\frac{1}{3}$		229 $\frac{2}{3} \frac{1}{3}$	253 $\frac{2}{3} \frac{1}{3}$
182.35		206 $\frac{1}{3}$		230 $\frac{2}{3} \frac{1}{3}$	254 $\frac{2}{3} \frac{1}{3}$
183.025		207 $\frac{2}{3} \frac{1}{3}$		231 $\frac{1}{3} \frac{2}{3} \frac{1}{3} \frac{1}{3}$	255 $\frac{1}{3} \frac{1}{3}$
184.3		208 $\frac{2}{3}$		232 $\frac{2}{3}$	256 $\frac{2}{3} \frac{1}{3}$
185.16		209 $\frac{2}{3} \frac{1}{3}$		233 $\frac{2}{3}$	257 $\frac{2}{3} \frac{1}{3}$
186.04		210 $\frac{1}{3} \frac{1}{3}$		234 $\frac{1}{3} \frac{2}{3}$	258 $\frac{2}{3} \frac{1}{3}$
187.1420+		211 $\frac{2}{3} \frac{1}{3}$		235 $\frac{2}{3} \frac{1}{3}$	259 $\frac{2}{3} \frac{1}{3}$
188.3		212 $\frac{2}{3}$		236 $\frac{2}{3} \frac{1}{3}$	260 $\frac{2}{3} \frac{1}{3}$
189.6		213 $\frac{2}{3}$		237 $\frac{2}{3}$	261 $\frac{2}{3} \frac{1}{3}$
190.1		214 $\frac{2}{3}$		238 $\frac{2}{3}$	262 $\frac{2}{3} \frac{1}{3}$
191.056		215 $\frac{2}{3}$		239 $\frac{2}{3}$	263 $\frac{2}{3} \frac{1}{3}$
192.625		216 $\frac{2}{3}$		240 $\frac{2}{3}$	264 $\frac{2}{3} \frac{1}{3}$
193.0768+		217 $\frac{2}{3}$		241 $\frac{2}{3}$	265 A's 850
194.2352+		218 $\frac{2}{3}$		242 $\frac{2}{3}$	265 B's 660
195.0476+		219 $\frac{2}{3}$		243 $\frac{2}{3}$	266 $\frac{2}{3}$ \$2,000
196.1538+		220 $\frac{2}{3}$		244 $\frac{2}{3}$	



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